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On conservative polynomials over $\mathbb{Q}$

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A polynomial $P \in \mathbb{C}[x]$ is called *conservative* iff its critical values are fixed: $P'(c) = 0 \Rightarrow P(c) = c$. A. Kostrikin introduced them in [1] and conjectured that the number of properly normalized conservative polynomials of degree n is equal to $\binom{2n-2}{n-1}$. Later D. Tischler [2] proved that the equivalence classes of conservative polynomials of degree $n+1$ correspond to plane trees with n edges.

F. Pakovich [3] pointed out that there is an action of the absolute Galois group Gal/$\mathbb{Q}$ on bicolored plane trees via its action on the corresponding conservative polynomials. It has an analogy in the theory of dessins d'enfants where an action of Gal/$\mathbb{Q}$ on plane trees comes from a Galois action on Shabat polynomials, which is a particular case of a more general Galois action on dessins (maps on orientable surfaces). The "geometric" Galois action on plane trees via dessins d'enfants turns out to be different from the "dynamical" action via conservative polynomials. F. Pakovich found three infinite series of conservative polynomials defined over $\mathbb{Q}$.

In this talk I will present new inifinite series of conservative polynomials (some of them are related to the Chebyshev polynomials $T_n(z)$), some sporadic examples of conservative polynomials with rational coefficients and will discuss dynamical fields of definitions of plane trees.

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Single-Image Quality Assessment: Recent Developments and Future Trends

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The digital imaging pipeline introduces various distortions, including those occurring during acquisition, processing, transmission, compression, storage, and reproduction. Addressing the question of how we can automatically and accurately predict image quality in a quantitative or perceived manner becomes crucial. Image quality assessment strives to provide computational models capable of automatically evaluating image quality. This keynote talk will delve into visual perception-driven image quality measurements. We will explore the underlying principles, discuss emerging trends, and showcase diverse applications. Additionally, we will present our recent research findings, offering a synopsis of the current state-of-the-art results in image quality measurements. Through this discussion, we will gain insights into the latest advancements and explore future trends in these technologies. Moreover, we will examine the potential commercial impact and the vast array of opportunities associated with perception-guided single-image quality assessment.

On the injectivity of the circular Radon transform, centered on a curve, on $\mathcal{C}(\mathbf{R}^2)$

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Thermoacoustic tomography (TAT) is a new safe method for Tomography (see [1], [2]). The mathematical base of TAT is the spherical Radon transform (SRT) that maps a function to its integrals over spheres with the centers at detectors.

The main mathematical problem is to restore a real valued function f supported compactly in a region $G \subset \mathbf{R}^d$ from the mean value $Mf(P,t)$ of f over spheres $S(P,t)$ of radius $t > 0$ centered on some hypersurface L.

The inversion of SRT is required in the mathematical models of thermo- and photo-acoustic tomography. Also, such inversions are of theoretical importance in many problems of integral geometry, approximation theory, functional analysis, and others [2]. Exact inversion formulas for SRT are known for different geometries of detectors.

It is known that a continuous, compactly supported within a body $D \subset \mathbf{R}^d$ (bounded and convex) function can be uniquely recovered from its spherical means over spheres centered on the boundary of D.

As usually, in literature the injectivity of SRT for compactly supported functions is considered and the situation becomes complicated without compactness of support ([1]).

The following problem arises: Let γ be a smooth curve. Find an additional condition so that it will possible to recover an unknown function using the circular Radon transform (CRT) with detectors placed on γ on the space of all continuous functions.

Theorem 1. *A smooth function $f \in \mathcal{C}^\infty(\mathbf{R}^2)$ can be reconstructed by its circular means Mf and the Fourier first cosine coefficientbe a_1 over circules with center on a smooth curve γ.*

In the case where γ is closed (the boundary of a convex domain), a reconstruction formula is found wiche has a local description. To reconstruct $f(\mathbf{x})$ at a point $\mathbf{x}$ we need the values of Mf and a_1 over circles with the centers in a neighborhood of $P \in \gamma$ and $0 < t \leq | P\mathbf{x} |$.

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On qualitative properties and asymptotic equivalence of quasy-linear differential equations with power-law nonlinearities

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We study the problem of asymptotic equivalence of the equations

$$y^{(n)} = 0 \tag{1}$$

and

$$y^{(n)} + \sum_{j=0}^{n-1} a_j(x) y^{(j)} = 0; \tag{2}$$

the equivalence of equation (2) and the equation

$$y^{(n)} + \sum_{j=0}^{n-1} a_j y^{(j)} + p(x)\,|y|^k \operatorname{sgn} y = 0; \tag{3}$$

the equivalence of equation (3) and the equation

$$y^{(n)} + \sum_{j=0}^{n-1} a_j(x) y^{(j)} + p(x)\,|y|^k \operatorname{sgn} y = f(x). \tag{4}$$

with $n \geq 2$, $k > 1$, and continuous functions p, f and $a_j, j = 1, ..., n-1$.

So, if an asymptotic equivalence of equations (i) and $(i+1)$, $i = 1, 2, 3$, is proved, we can describe the asymptotic behavior of solution to $(i+1)$ with the help of asymptotic behavior of solutions to (i) and vice versa. Previous results are formulated in [1]–[5]. See also the references therein.

Theorem 1. *Suppose that continuous functions p, f are bounded while $a_0, \ldots, a_{n-1}$ are continuous functions satisfying the inequalities*

$$\int_{x_0}^{\infty} x^{n-j-1} |a_j(x)|\, dx < \infty, \ j \in \{0, \ldots, n-1\}, \tag{5}$$

and the function f satisfies the condition

$$|f(x)| \leq Ce^{-\gamma x}, \ C > 0, \ \gamma > 0,$$

Then for any solution $y(x)$ to equation (3) tending to zero as $x \to \infty$, there exists a solution $z(x)$ to equation (4) such that

$$|y(x) - z(x)| = O(e^{-\gamma x}), \quad x \to \infty. \tag{6}$$

Similarly, for any solution $z(x)$ to equation (4) tending to zero as $x \to \infty$, there exists a solution $y(x)$ to equation (3) satisfying (6).

Theorem 2. *Suppose $Y > 0$, $\alpha = \frac{n}{k-1}$, $a_0, \ldots, a_{n-1}$, p, f are continuous functions, p is bounded and condition (5) holds. There exists $\sigma > \alpha$ such that if*

$f = o(x^{-\sigma-n})$ at infinity and y is a solution of (3) *satisfying $|y(x)| \leq Yx^{-\alpha}$, then there exists a unique solution $z(x)$ to equation* (4) *such that*

$$|y(x) - z(x)| = O(x^{-\sigma}), \quad x \to \infty.$$

And vice versa.

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On the spread of infinite groups

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The spread of a group G, written $s(G)$, is the greatest k such that for any non-trivial elements $x_1, \ldots, x_k$ of G there exists $y \in G$ satisfying the conditions $G = \langle x_i, y \rangle$ for all $i = 1, 2, \ldots, k$. This notion was first introduced in [1] in 1975. In 2008 [2] it was shown that $s(G) \geq 2$ for any non-abelian finite simple group G. In 2019, Donoven and Harper [3] proved that $S(G) \geq 1$ for all Higman–Thompson and Brin–Thompson groups G. A subset S of a group G is a totally dominating set for G if for any non-trivial element $x \in G$ there exists $y \in S$ such that $G = \langle x, y \rangle$ (see [3]). Tarski's monsters are infinite simple groups, any pair of non-commuting elements of which is a totally dominating set. It is clear that the infinite cyclic group $\mathbb{Z}$ also has a finite totally dominating set. In [3] the authors posed the following questions:

1. *Is there an infinite group, other than $\mathbb{Z}$ or the Tarski monsters, that has a finite totally dominating set* (see Question 5 [3]);

2. *Is there an infinite group, other than $\mathbb{Z}$ or the Tarski monsters, for which* $s(G) = \infty$ (see Question 3 [3]).

We have proved that both these questions have a positive answer.

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Some extensions of Marcinkiewicz–Zygmund inequalities in the real ball

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Over the unit ball B in $\mathbb{R}^n$ $(n \geq 2)$, we give some extensions and generalizations for classical Marcinkiewicz–Zygmund inequality on majorization of Littlewood–Paley g-function for harmonic functions.

Elements (functions) that are universal with respect to a minimal system

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We call an element U conditionally universal for a sequential convergence space $\mathbf{\Omega}$ with respect to a minimal system $\{\varphi_n\}_{n=1}^{\infty}$ in a continuously and densely embedded Banach space $\mathcal{X} \hookrightarrow \mathbf{\Omega}$ if the partial sums of its phase-modified Fourier series is dense in $\mathbf{\Omega}$. We will call the element U almost universal if the change of phases (signs) needs to be performed only on a thin subset of Fourier coefficients. In this paper we prove the existence of an almost universal element under certain assumptions on the system $\{\varphi_n\}_{n=1}^{\infty}$. We will call a function U asymptotically conditionally universal in a space $L^1(\mathcal{M})$ if the partial sums of its phase-modified Fourier series is dense in $L^1(F_m)$ for an ever-growing sequence of subsets $F_m \subset \mathcal{M}$ with asymptotically null complement. Here we prove the existence of such functions U under certain assumptions on the system $\{\varphi_n\}_{n=1}^{\infty}$. Moreover, we show that every integrable function can be slightly modified to yield such a function U.

In particular, we establish the existence of almost universal functions for $L^p([0,1])$, $p \in (0,1)$, and asymptotically conditionally universal functions for $L^1([0,1])$, with respect to the trigonometric system.

This is a joint work with M. Grigoryan and M. Ruzhansky.

On a Dirichlet Problem for Higher Order Properly Elliptic Equation in the Class of Continuous Functions

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Let $D = \{z : |z| < 1\}$ be a unit disk and $\Gamma = \partial D$ its boundary. We consider the higher order properly elliptic differential equation

$$\sum_{k=0}^{2N} A_k \frac{\partial^{2N} u}{\partial x^k \partial y^{2N-k}}(x,y) = 0, \quad (x,y) \in D, \tag{1}$$

where A_k are such complex constants ($A_0 \neq 0$), that the numbers λ_j ($j = 1, \ldots, 2N$) – the roots of characteristic equation, satisfy the condition

$$\Im\lambda_j > 0, \;\; j = 1, \ldots, N; \;\; \Im\lambda_j < 0, \; j = N+1, \ldots, 2N. \tag{2}$$

The solution of the equation (1) we seek in the class $C^{2N}(D) \bigcap C^{N-1}(\overline{D})$, and on the boundary Γ ($z = e^{i\theta}$) satisfy Dirichlet conditions:

$$\left.\frac{\partial^j u}{\partial N^j}\right|_{\Gamma} = f_j(\theta), \;\; j = 0, N-1; \tag{3}$$

where $f \in C^{N-1-j}(\Gamma)$ are the given functions.

In the classic formulation of the Dirichlet problem the boundary functions are continuous up to the boundary, but as the Cauchy type integral is not invariant in the space of the continuous functions, investigation of the problem in this space is difficult. In the works [1], [2], [3], using the modifications of the Cauchy kernel, the Dirichlet problem in the class of continuous functions, was investigated for the second order elliptic operators. Prof. H. M. Hairapetyan introduced the new formulation of the boundary value problems, which helps him and his students to solve the Dirichlet problem for some model higher order equations in the space of continuous functions ([4]). In the talk, using Abel summation of the Fourier series, we shall see that in the unit disk the results of the paper [5] (where the problem (1), (3) was considered for $f \in C^{(N-1-j,\sigma)}(\Gamma)$) are valid for the problem (1), (3).

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Optimization and control of transport flows on Wardrop ptimal networks: An evolutionary game dynamics approach

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The network optimization problem is the topic of active research nowadays, especially, in optimal transport theory and communicaton networks. In 1952 John Wardrop formulated two principles of optimality of flow distributions in networks that describe the *user equilibrium* and the *system optimum*. The first Wardrop principle describes an optimal flow distribution across alternative parallel links in the network, namely, it states that *the effective costs of all utilized links are equal and less than the effective costs of those unutilized links* for every fixed source-destination pair, while the system optimum is the optimal distribution of the flow for which the average effective cost for all used links is minimal [1, 2].

The fundamental problem at the intersection of transportation theory and game theory is to quantify the inefficiency of system's performance. The degradation in a system's efficiency due to selfish non-cooperative behavior is called the *price of anarchy*. In the context of transport networks, the price of anarchy is defined as the ratio of average travel time between the selfish users' equilibrium and the system optimum. The study [5] shows that for transport networks with affine cost functions the price of anarchy is at most 4/3. The price of anarchy is at least 1 by definition, and its proximity to 1 indicates that the user equilibrium is approximately socially optimal, thereby implying that the effects of selfish behavior are relatively benign. In fact, all the studies deal only with approximations or estimations of the price of anarchy for some typical transport networks under various cost objectives.

In this work, we introduce and study the so-called *Wardrop Optimal Network* in which the selfish behavior of non-cooperative network users has a negligible impact on network performance degradation. The *Wardrop Optimal Flow*, that exists in any Wardrop Optimal Network, satisfies both principles – the user (or Wardrop) equlibrium and the system optimum. In other words, the Wardrop optimal flow is achieved when each user in the system acts selfishly and seeks to minimize his individual travel time, while the network as a whole remains free of any traffic congestion or inefficiency. The introduced *Wardrop Optimal Networks are the only networks for which the price of anarchy equals exactly* 1, which is extremely desirable property because the pattern chosen by the user acting in his own self interest coincides with the pattern optimal for the system. We present a characterization of Wardrop optimal flows and provide a geometric description of the set of all Wardrop optimal networks with common Wardrop optimal flow. We investigate dynamic properties of Wardrop optimal networks and examine the Wardrop optimal flows on dynamic networks in which link latency (cost) functions change over time, i.e., at each next time instant (iteration, observation) the functions may differ from those at the previous step. We propose a new dynamical model of optimal flow distribution using the ideas of evolutionary game theory [6, 7] by presenting a discrete-time replicator equation on Wardrop optimal network, and describe the dynamics, equilibrium and stability conditions of the replicator equation, for which the Nash equilibrium, the Wardrop equilibrium, and the system optimum are the same flow distribution of the network. If time permits, some future research will be outlined.

(This is a joint work with A. Kalampakas and M. Saburov)

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On semigroup automorphisms of square matrices

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Let $M_n(\mathbb{F}_p)$ denote the multiplicative semigroup of square matrices of size n over the prime field of order p.

In this talk, we consider the problem of description of the automorphisms of $M_n(\mathbb{F}_p)$. The automorphisms of these matrix semigroups have been thoroughly explored since 1950s. We introduce a novel approach for this problem, which is quite elementary, being based on matrix operations.

In particular, we show that all automorphisms of $M_n(\mathbb{F}_p)$ are inner.

Density of quantized approximations

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The aim of my talk is to present and discuss the following result.

Theorem 1. *Let X, Y be Hilbert spaces. Let $E \subset \mathbb{R}^2$ be a connected compact set, $f : E \to X$ be a Lipschitz map, $A : X \to Y$ be a compact operator. If $A(f(E))$ does not lie in any half-space of Y, then the sums*

$$y_1 + \cdots + y_n, \qquad y_k \in A(f(E)), n \in \mathbb{N}$$

are dense in Y.

In particular, I'll show how this general result entails the famous

Theorem 2 (Korevaar's theorem). *For every bounded simply connected domain $D \subset \mathbb{C}$, the sums*

$$\sum_{k=1}^{n} \frac{1}{z - a_k}, \qquad a_k \in \partial D, n \in \mathbb{N}$$

are dense in the space $A(D)$ of functions holomorphic in D, with a topology of uniform convergence on compact subsets of D.

This is a joint work with K. S. Shklyaev.

Visualization of extremely sparse contingency tables by taxicab correspondence analysis: A case study

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In this talk, we present an overview of taxicab correspondence analysis, a robust variant of correspondence analysis, for visualization of sparse contingency tables. In particular to visualize an extremely sparse textual data set of size 590 by 8260 concerning fragments of 8 sacred books recently introduced by Sah and Fokoué (2019) and studied quite in detail by (12 + 1) dimension reduction methods (t-SNE, UMAP, PHATE,...) in machine learning by Ma, Sun and Zou (2022, 2023).

Deviation Identities on Solutions for a Class of the Obstacle Problems

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Let us consider a variational problem generated by a differential operator Λ :

$$J(v) = \frac{1}{2}\,(B\cdot\Lambda v, \Lambda v)_Y - (f, v)_{L_2} \to \min_{\mathbb{K}}. \tag{1}$$

Here $\Lambda\colon V \to Y$, $V = \left\{v \in H^k(\Omega) : v\big|_{\partial\Omega} = 0\right\}$, $\Omega \subset \mathbb{R}^d$ is a bounded simply connected domain with Lipschitz continuous boundary $\partial\Omega$, and $f \in L_2(\Omega)$.

Further, $Y = M^{d\times d}_{sym}(L_2(\Omega))$ is a space of symmetrical function-valued matrices with scalar product $(q,g)_Y = \int\limits_\Omega q : g\,dx$, where $q : g = \sum\limits_{i=1}^{d}\sum\limits_{j=1}^{d} q_{ij}g_{ij}$. We consider $B = \left\{b_{ijkl}\right\}$ with certain (physically grounded) properties and executing a linear continuous mapping from Y to itself, $B\cdot\rho = \sum\limits_{k=1}^{d}\sum\limits_{l=1}^{d} b_{ijkl}\rho_{kl}$ $(\rho_{kl} \in Y)$.

Finally, $\mathbb{K} = \{v \in V :\ v \geqslant \varphi \text{ a.e. in } \Omega\}$. Function $\varphi \in C^2(\overline{\Omega})$ such that $\varphi \leqslant 0$ on $\partial\Omega$ is called an obstacle.

Our aim is to get deviation identity for approximate solutions v of problem (1). The left-hand side of the desired identity consists of two parts. The first one involves terms decribing distance between exact and approximate solution of problem (1) and its dual. The second one is an additional adjusting expression depending on the obstacle φ.

The right-hand side of the identity is the measure of distance only between approximate solutions of primal and dual problems, thus, is fully computable. Note also that the mentioned deviation identity does not depend on the method by which approximate solutions were obtained. Therefore, it allows to conclude not only how efficent is a certain approximation of solution but also which of approximations works better.

Problem (1) can refer to various applications due to multiple options for choosing Λ and B. For example, it can be utilised in the elasticity theory and hydrodynamics, see [1] and bibliography therein. Main concepts that we use here are the duality theory and the so called error identity (see [1, 2] for more details).

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The Structure of Distributive d-algebras

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The classification of left and right distributive hyperidentities in binary algebras with some invertible operation (q-algebras) is obtained in [1]. In [2] the Belousov [4] - Aczel [5] problem on distributivity criterion for invertible algebras and q-algebras is solved (see also [3]). In this talk we present more general results, i.e. characterization theorems for the distributivity of binary algebras with some divisible operation (d-algebras).

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On the L^p - greedy universal functions with respect to the generalized Walsh system

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Let $a \geq 2$ be a fixed integer, and let $\omega_a = e^{2\pi i/a}$.

The generalized Rademacher system of order a is defined as follows (see [1]):

For $n = 0$, let

$$\varphi_0(x) = \omega_a^k \qquad x \in \left[\frac{k}{a}, \frac{k+1}{a}\right), \quad k = 0, 1, \ldots, a-1,$$

then for any $n \geq 1$

$$\varphi_n(x+1) = \varphi_n(x) = \varphi_0(a^n x).$$

We denote the generalized Walsh system of order a as Ψ_a. Note that Ψ_2 is the classical Walsh system, and the system Ψ_a is a special case of the Vilenski system.

For a given function $f \in L^p[0,1)$, where $p \geq 1$, we denote the Fourier coefficients of f with respect to the system Ψ_a as $c_k(f)$. The spectrum of a function $f(x)$ (denoted as $spec(f)$) is the set of indices for which the coefficients $c_k(f)$ are nonzero, i.e., $spec(f) = \{k \in N, c_k(f) \neq 0\}$.

Definition. The m-th greedy approximant of an element $f \in L^p[0,1)$, $p \geq 1$, with respect to the system Ψ_a is defined as the following sum:

$$G_m(f, \phi) = \sum_{k \in \Lambda} c_k(f) \phi_k, \tag{1}$$

where $\Lambda \subset 1, 2, \ldots$ is an arbitrary set of indices of cardinality m that satisfies the condition: $|c_n(f)| \geq |c_k(f)|$ if $n \in \Lambda$, $k \notin \Lambda$.

We say that the greedy algorithm for a function $f \in L^p[0,1]$, $p \geq 0$, converges with respect to Ψ_a if the sequence $G_m(x, f)$ converges to $f(t)$ in the L^p norm.

There have been many studies on the convergence of the greedy algorithm using different systems. In particular, T. V. Kӧrner constructed a function from L^2 (later also continuous) for which the greedy algorithm diverges almost everywhere with respect to the trigonometric system. Furthermore, V. N. Temlyakov constructed an example of a function $f \in L^p$, $p \in [1, 2)$ (respectively, $p > 2$), for which the greedy algorithm diverges in measure (in the L^p norm, $p > 2$), with respect to the trigonometric system.

In the work:

S.A. Episkoposian, M.G. Grigorian, *L^p-convergence of the greedy algorithm by the generalized Walsh system,* Journal of Mathematical Analysis and Applications, Vol. 389, No. 2, 2012, pp. 1374-1379.

the following statement is proven:

Theorem 1. *Let $p > 2$. Then, for any $\varepsilon > 0$ and $f \in L^p[0, 1)$, there exists a function $g \in L^\infty[0, 1)$ such that $|x \in [0, 1) : g \neq f| < \varepsilon$ and the greedy algorithm for the function g with respect to the system Ψ_a, $a \geq 2$, converges uniformly on $[0, 1)$.*

In this work, we strengthen Theorem 1 and prove the following theorem:

Theorem 2. *There exists a function $U \in L^1[0, 1)$ with the following property: for any $0 < \epsilon < 1$, $p > 1$, and any function $f \in L^p[0, 1)$, $p \geq 1$, we can find a function $\tilde{f} \in L^\infty[0, 1)$ such that $|x \in [0, 1) : \tilde{f} \neq f| < \epsilon$, the greedy algorithm for the function $\tilde{f}$ with respect to the system Ψ_a, $a \geq 2$, converges in $L^p[0, 1)$, and $|c_k(\tilde{f})| = c_k(U)$ for all $k \in spec(\tilde{f})$.*

Additionally, the following statement holds:

Theorem 3. *There exists a function $U \in L^1[0, 1)$ with the following property: for any $0 < \epsilon < 1$ and any function $f \in \bigcap_{p>1} L^p[0, 1)$, we can find a function $\tilde{f} \in \bigcap_{p>1} L^p$ such that $|x \in [0, 1) : \tilde{f} \neq f| < \epsilon$, the greedy algorithm for the function $\tilde{f}$ with respect to the system Ψ_a, $a \geq 2$, converges in all spaces $L^p[0, 1)$, $p > 1$, and $|c_k(\tilde{f})| = c_k(U)$ for all $k \in spec(\tilde{f})$.≥ 2.*

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On two systems of nonlinear partial differential equations

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Numerous scientific works, monographs, and textbooks are devoted to the research of the system of nonlinear partial differential equations (SNPDE). One model of such type describing the process of electromagnetic field penetration in the substance is a well-known system of Maxwell equations [1]:

$$\frac{\partial H}{\partial t} = -rot(\nu_m rot H), \tag{1}$$

$$\frac{\partial \Theta}{\partial t} = \nu_m (rot H)^2, \tag{2}$$

where $H = (H_1, H_2, H_3)$ is a vector of the magnetic field, Θ is temperature, v_m characterizes the electro-conductivity of the substance. As a rule, these coefficients are functions of the argument Θ. Equations (1) describe the process of diffusion of the magnetic field while equation (2) expresses the change of the temperature at the expense of Joule heating. Many important processes are described applying the abovementioned Maxwell's system (see, e.g., [2], [3] and references therein).

System (1), (2) does not take into account many physical effects. For a more thorough description, first of all it is desirable to take into consideration heat conductivity. In this case together with (1) instead of (2) the following equation is considered [1]

$$\frac{\partial \Theta}{\partial t} = \nu_m (rotH)^2 + div(k_m grad\Theta), \tag{3}$$

where k_m is a coefficient of heat conductivity. This coefficient is a function of Θ as well.

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Efficient verification of some properties of finite quasigroups

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Definition 1. *A finite quasigroup is a finite set Q endowed with a binary operation f such that for any $a, b \in Q$ the equations $f(x, a) = b$ and $f(a, y) = b$ are uniquely solvable.*

Hereinafter we consider only finite structures, so for the sake of brevity the word "finite" will be omitted.

Quasigroups are a promising platform for implementation of various cryptographic algorithms (see, e.g., the review [1]). Cryptographic requirements impose a number of restrictions on quasigroups used.

Definition 2. *A quasigroup (Q, f) is affine if there exists an Abelian group $G = (Q, +)$, $\alpha, \beta \in Aut(G)$ and $c \in Q$ such that $f(x, y) \equiv \alpha(x) + \beta(y) + c$.*

Non-affinity is the crucial factor from the viewpoint of algebraic attacks, since solvability of a system of equations over a quasigroup can be decided with polynomial complexity if and only if the quasigroup is affine (otherwise the problem is NP-complete; see [2]).

Definition 3. *A quasigroup is simple if it has only trivial congruences.*

Non-simplicity can obviously be used to split a problem for a quasigroup into a subproblem on the set of equivalent classes and subproblems for individual equivalence classes.

Definition 4. *A quasigroup (Q, f) has a proper subquasigroup if there exists a non-empty set $H \subsetneq Q$ closed under f.*

The existence of a proper subquasigroup can lead to a fixed point (if $|H| = 1$) or to reduction of the operation to an essentially smaller set, as $|H| \leq |Q|/2$.

In [3, 4] we introduced algorithms for deciding non-affinity, simplicity and the existence of subquasigroups. The following assertion improves the upper bounds on algorithm complexities.

Theorem 1. *Non-affinity, simplicity and the existence of proper subquasigroups for a quasigroup (Q, f) can be decided with the complexity $O\left(|Q|^2 \log |Q|\right)$, $O\left(|Q|^3\right)$, $O\left(|Q|^{7/3}\right)$ respectively.*

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Proper families of functions and fixed point networks

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Suppose that $k, n \in \mathbb{N}$, $E_k = \{0, 1, \ldots, k-1\}$, P_k^n is the set of all n-ary functions on E_k.

Definition 1. *A functional network of size n is a tuple $(f_1, \ldots, f_n)$, where $f_i \in P_k^n$, $1 \leq i \leq n$.*

Let F be a functional network. Then G is a subnetwork of F, if G can be obtained from F by substituting constants for some of the variables and cancelling functions corresponding to the fixed variables.

Functional network is a natural model for various processes; e.g., a model of a gene network: variables can be interpreted as gene expressions, and functions — as regulation rules. Of special interest in this context are fixed points of the network.

Definition 2. *A functional network F is a hereditarily unique fixed point network (HUFP) if F and any subnetwork of F has a unique fixed point.*

HUPF networks were studied for the case $k = 2$ in a series of papers (see, e.g., [1, 2]), and a number of criteria for HUPF were established. To the best of our knowledge, the case $k \geq 3$ remains uninvestigated.

Definition 3. *A functional network $(f_1, \ldots, f_n)$ is a proper family if for any $\alpha, \beta \in E_k^n$, $\alpha \neq \beta$, there exists an index i such that $\alpha_i \neq \beta_i$ and $f_i(\alpha) = f_i(\beta)$.*

Proper families were introduced by Nosov for the purpose of memory-efficient specification of large parametric classes of quasigroups (see, e.g., [3]).

We say that a functional network $G = (g_1, \ldots, g_n)$ is a reencoding of a functional network $F = (f_1, \ldots, f_n)$, if $g_i(x_1, \ldots, x_n) = \tau_i(f_i(\sigma_1(x_1), \ldots, \sigma_n(x_n)))$, $i = 1, \ldots, n$, for some permutations $\sigma_1, \ldots, \sigma_n$ and $\tau_1, \ldots, \tau_n$ on E_k; G is obtained from F via a consistent variable renumbering if $g_i(x_1, \ldots, x_n) = f_{\delta(i)}(x_{\delta(1)}, \ldots, x_{\delta(n)})$, where δ is a permutation on $\{1, \ldots, n\}$.

Theorem 1. *If $k = 2$, then a functional network is HUPF if and only if it is a proper family. If $k \geq 3$, then any proper family is a HUPF network; if a functional network and all its reencodings are HUPF, then this network is a proper family.*

Reencodings play a special role for proper families of functions.

Theorem 2. *Suppose that $k \geq 3$, $n \geq 2$, Consider two sets of bijections $\mathbb{A}, \mathbb{B}$ on E_k^n such that for any $A \in \mathbb{A}$, $B \in \mathbb{B}$ and any proper family F it holds that $A(F(B(x)))$ is also proper. Then A and B are superpositions of a consistent variable renumbering and a reencoding.*

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Uniqueness of Ciesielski series with subsequence of partial sums converging to an integrable function

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Some uniqueness theorems for series in the Ciesielski system are proved in this work. In particular, if the partial sums $S_{n_i}(x) = \sum_{n=-k+2}^{n_i} a_n f_n(x)$ of the Ciesielski series $\sum_{n=-k+2}^{\infty} a_n f_n(x)$ converge in measure to an integrable function f and $\sup_i |S_{n_i}(x)| < \infty$ when $x \notin B$, where B is some countable set, with $S_{n_i}(x)/n_i \rightrightarrows 0$ on [0,1] and $\sup_i \frac{n_{i+1}}{n_i} < \infty$, then this series is the Fourier-Ciesielski series of the function f.

On the Prediction Error for Singular Stationary Processes

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Let $X(t)$, $t \in \mathbb{Z} := \{0, \pm 1, \ldots\}$, be a centered second-order stationary process with spectral density $f(\lambda)$, $\lambda \in [-\pi, \pi]$. Let $\sigma^2(f)$ denote the best linear prediction error of the random variable $X(0)$ by the entire infinite past: $\{X(t),\ t \le -1\}$, and let $\sigma_n^2(f)$ be the linear prediction error by a finite past of length n: $X(-n), \ldots, X(-1)$.

From the prediction point of view it is natural to distinguish the class of processes for which we have *error-free prediction* by the entire infinite past, that is, $\sigma^2(f) = 0$. Such processes are called *singular* or *deterministic*. Processes for which $\sigma^2(f) > 0$ are called *regular* or *nondeterministic*.

Define the *relative prediction error* $\delta_n(f) := \sigma_n^2(f) - \sigma^2(f)$, and observe that $\delta_n(f) \ge 0$ and $\delta_n(f) \to 0$ as $n \to \infty$. But what about the speed of convergence of $\delta_n(f)$ to zero as $n \to \infty$? This speed depends on the regularity nature (regular or singular) of the observed process $X(t)$.

It turns out that for regular processes the asymptotic behavior of the prediction error is determined by the dependence structure of the observed process $X(t)$ and the differential properties of its spectral density f, while for singular processes it is determined by the geometric properties of the spectrum of $X(t)$ and singularities of its spectral density f.

The prediction problem we are interested in is *to describe the rate of decrease of* $\delta_n(f)$ *to zero as* $n \to \infty$, depending on the regularity nature of the observed process $X(t)$.

In this talk, we discuss the above problem for the less investigated case - singular processes. We provide extensions of Rosenblatt's [7] and Davisson's [6] results concerning asymptotic behavior and upper bounds for the finite prediction error $\sigma_n^2(f)$. Examples illustrate the stated results.

The talk is based on the joint works ([1]–[5]) with Nikolay Babayan (Russian-Armenian University) and Murad Taqqu (Boston University).

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Fractional Order Orlicz-Sobolev space defined on Metric Measure Spaces

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In this talk, we defined fractional order Orlicz-Sobolev spaces on metric measure space. We give equivalent characterization using so call fractional order Hajłasz gradient.

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On the Boundedness and Convergence Properties of some Operator Sequences Associated with Walsh System

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A certain sequence of operators associated with Walsh functions will be discussed. A necessary and sufficient condition will be proved for a given sequence of operators to be convergent almost everywhere for every integrable function.

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Machine learning architectures for price formation models with common noise

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We propose a machine learning method to solve a mean-field game price formation model with common noise. This involves determining the price of a commodity traded among rational agents subject to a market clearing condition imposed by random supply, which presents additional challenges compared to the deterministic counterpart. Our approach uses a dual recurrent neural network architecture encoding noise dependence and a particle approximation of the mean-field model with a single loss function optimized by adversarial training. We provide a posteriori estimates for convergence and illustrate our method through numerical experiments.

Universal functions with respect to the trigonometric system

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The existence of functions and series that are universal in some sense in different classes of functions was studied by many mathematics, and publications on this topic appear regularly. The first examples of universal functions were constructed by Birkhoff within the framework of complex analysis, with entire functions represented in any circle by uniformly convergent shifts of a universal function, and by Martsinkevich within the framework of real analysis, with any measurable function represented as the limit almost everywhere of some sequence of difference relations of a universal function It follows from Kolmogorov's theorem (the Fourier series of each integrable function in the trigonometric system converges in measure) that there is no integrable function whose Fourier series in the trigonometric system is universal in the class of all measurable functions. Getsadze proved that an analog of Kolmogorov's theorem does not hold for the double trigonometric system. Despite this recently S. V. Konyagin [1] has proved that there is no function universal for the space $L^p(T^2)$, $p \in (0,1)$., with respect to the double trigonometric system (see also [2]). More exactly, the following theorem is true.

Theorem 1 (Konyagin)**.** *There is no function $f \in L^1(T^d)$, such that the rectangular partial sums of its multiple trigonometric Fourier series are dense in $L^p(T^d)$, $p \in (0,1)$.*

Nevertheless that in [3] author proved(see also [4]).

Theorem 2. *There exists a function that is almost universal both over rectangles and over spheres for the class $L^p(T^d)$, $p \in (0,1)$, with respect to the multiple trigonometric system.*

Note tha this property possesses the majority of functions. The following stronger theorem holds (see [2])

Theorem 3. *For any $\varepsilon > 0$ exists a measurable set $E \subset T$ with $|E| > 1 - \varepsilon$, so that for each function $f \in L^1(T^d)$ one can find such a function $g \in L^1(T^d)$ coinciding with f on E, wich is almost universal both over rectangles and over spheres for the class $L^p(T^d)$, $p \in (0,1)$, with respect to the multiple trigonometric system.*

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Can 5th Generation Local Training Methods Support Client Sampling? Yes!

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The celebrated algorithm of [1] is based on three components: client sampling (CS), data sampling (DS) and local training (LT). While the first two are reasonably well understood, the third component, whose role is to reduce the number of communication rounds needed to train the model, resisted all attempts at a satisfactory theoretical explanation. [3] identified four distinct generations of LT methods based on the quality of the provided theoretical communication complexity guarantees. Despite a lot of progress in this area, none of the existing works were able to show that it is theoretically better to employ multiple local gradient-type steps (i.e., to engage in LT) than to rely on a single local gradient-type step only in the important heterogeneous data regime. In a recent breakthrough embodied in their method and its theoretical analysis, [2] showed that LT indeed leads to provable communication acceleration for arbitrarily heterogeneous data, thus jump-starting the 5$^{\text{th}}$ generation of LT methods. However, while these latest generation LT methods are compatible with DS, none of them are able to support CS. In this work we resolve this open problem in the affirmative. In order to do so, we had to base our algorithmic development on new algorithmic and theoretical foundations.

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Matrix majorizations: their properties and applications

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Vector majorization is a classical notion useful in many areas of mathematics and their applications. There are many ways to generalize vector majorization to real matrices. Different types of matrix majorizations have been applied to different areas of research. For example, directional majorization is an important generalization of the vector case and can be also applied in Economics, while row-stochastic majorization plays an important role in the theory of statistical experiments. We introduce a new majorization order for sets of matrices which generalizes several existing notions of matrix majorization. The motivation to study this majorization concept comes from mathematical statistics and involves the information content of experiments. We investigate properties of this new order both of algebraic and geometric character. In particular, we establish results on so-called minimal cover classes with respect to the introduced majorization. We also obtain the complete characterization for the matrix mappings preserving or converting majorizations.

The talk is based on a series of joint works with P. Shteyner and G. Dahl.

Periodic orthonormal spline systems as bases and unconditional bases in $H^1(\mathbb{T})$

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We give a geometric characterization of knot sequences (s_n), for which the corresponding periodic orthonormal spline system of arbitrary order k, $k \in \mathbb{N}$, is a basis in the atomic Hardy space on the torus $H^1(\mathbb{T})$. Furthermore, we obtain geometric characterization of knot sequences (s_n) for which that system forms an unconditional basis in $H^1(\mathbb{T})$.

Quasisymmetric Uniformization of countably connected Metric Surfaces

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A metric space X is *Ahlfors 2-regular* if 2-dimensional Hausdorff measure of a ball of radius r in X is comparable to r^2: $\mathcal{H}^2(B(x,r)) \asymp r^2$.

A metric space X is of *bounded turning* if there is a constant $C \geq 1$ such that for all $x, y \in X$ there is a continuum $\gamma_{x,y}$ connecting x and y such that $\mathrm{diam}(\gamma_{x,y}) \leq C\mathrm{dist}(x, y)$.

A domain W of the Riemann sphere $\hat{\mathbb{C}}$ is called a *circle domain* if every boundary component of W is either a point or a round circle.

Theorem 1 (H.-Rehmert, 2022)**.** *Let X be a metric space homeomorphic to a countably connected domain in $\hat{\mathbb{C}}$. Suppose X is Ahlfors 2-regular and is of bounded turning. Then X is quasisymmetric to a circle domain $W \subset \hat{\mathbb{C}}$ with uniformly relatively separated boundary circles if and only if*

- *$\overline{X}$ is compact and*
- *X has 2-transboundary Loewner property.*

The notion of 2 transboundary Loewner property is a quasisymmetricaly invariant property of a metric space which is introduced in our work and is a version of the well known Loewner property of Heinonen and Koskela for Schramm's transboundary modulus.

Theorem 1 provides an extension of Bonk's celebrated uniformization of planar Sierpinski carpets [1] to the non-planar setting, and also generalizes several of recent results about quasisymmetric uniformization of finitely and countably connected metric surfaces due to Merenkov-Wildrick [2] and Rajala-Rasimus [3].

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On n-independent, n-correct and GC_n sets

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In our talk, we consider some recent results on n-correct, n-independent, GC_n sets, and the Gasca-Maeztu conjecture. The results are joint with V. Bayramyan, A. Kloyan, S. Toroyan, V. Vardanyan, N. Vardanyan, G. Vardanyan and D. Voskanyan, respectively.

Denote the space of all bivariate polynomials of total degree at most n by Π_n. A planar set $\mathcal{X}$ is called n-correct if the interpolation problem with Π_n and arbitrary data is unisolvent. For n-correct sets we have that $\#\mathcal{X} = \binom{n+2}{2}$. A set $\mathcal{X}$ is called n-independent if each node has a fundamental polynomial from Π_n. An n-correct set $\mathcal{X}$ is called GC_n set if the fundamental polynomial of each node is a product of n linear factors (as in the univariate case).

At most $n+1$ nodes can be collinear in any n-independent set and a line passing through $(n+1)$ nodes is called a maximal line. We say that a node A in an n-correct set uses a line ℓ if ℓ divides the fundamental polynomial of A.

The Gasca-Maeztu conjecture (1982) states that every GC_n set has a maximal line. Until now the conjecture has been proved only for the cases $n \leq 5$.

For an n-correct or GC_n set $\mathcal{X}$ we consider the properties of maximal lines, n-node lines, and the subset of nodes in $\mathcal{X}$ that use a given line.

At the end, in terms of n-independent sets, we bring a necessary and sufficient condition, for a set of mn points to be the set of intersection points of some two algebraic curves of degree m and n respectively (a joint result with D. Voskanyan).

A Second-Order Optimization Method with Low-Rank Updates and Global $\mathcal{O}(1/k^2)$ Convergence Rate

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In this paper, we propose the first sketch-and-project Newton method with fast $\mathcal{O}\left(k^{-2}\right)$ global convergence rate for self-concordant functions. Our method, SGN, can be viewed in three ways: i) as a sketch-and-project algorithm projecting updates of Newton method, ii) as a cubically regularized Newton method in sketched subspaces, and iii) as a damped Newton method in sketched subspaces. SGN inherits best of all three worlds: cheap iteration costs of sketch-and-project methods, state-of-the-art $\mathcal{O}\left(k^{-2}\right)$ global convergence rate of full-rank Newton-like methods and the algorithm simplicity of damped Newton methods. Finally, we demonstrate its comparable empirical performance to baseline algorithms.

Information-theoretic investigation of some security systems

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One of the problems of information - theoretic security [1] concerns secure communication over a wiretap channel [2]. The aim in the general wiretap channel model is to maximize the rate of the reliable communication from the source to the legitimate receiver, while keeping the confidential information as secret as possible from the wiretapper. In [3] E - capacity - equivocation region and the E - secrecy capacity function for the wiretap channel were introduced and investigated, which are, correspondingly, the generalizations of the capacity - equivocation region and secrecy - capacity studied by Csiszár and Körner [4]. The E - capacity - equivocation region is the closure of the set of all achievable rate - reliability and equivocation pairs, where the rate - reliability function represents the optimal dependence of rate on the error probability exponent (reliability). By analogy with the notion of E - capacity [5, 6], we consider the E - secrecy capacity function that for the given E is the maximum rate at which the message can be transmitted being kept perfectly secret from the wiretapper. Next step is the investigation of E - secrecy capacity of the compound wiretap channel model, when the channels to the legitimate receiver and to the wiretapper depends on the number of possible states. The upper and lower bounds on the secrecy capacity of this model were derived in [7].

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Safe Learning and Control with L1 Adaptation

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Learning-based control paradigms have seen many success stories with various robots and co-robots in recent years. However, as these robots prepare to enter the real world, operating safely in the presence of imperfect model knowledge and external disturbances is going to be vital to ensure mission success. In the first part of the talk, we present an overview of L1 adaptive control, how it enables safety in autonomous robots, and discuss some of its success stories in the aerospace industry. In the second part of the talk, we present some of our recent results that explore various architectures with L1 adaptive control while guaranteeing performance and robustness throughout the learning process. An overview of different projects at our lab that build upon this framework will be demonstrated to show different applications.

Accelerated Adaptive Cubic Regularized Quasi-Newton Methods

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In this paper, we propose the first Quasi-Newton method with a global convergence rate of $O(k^{-1})$ for general convex functions. Quasi-Newton methods, such as BFGS, SR-1, are well-known for their impressive practical performance. However, they may be slower than gradient descent for general convex functions, with the best theoretical rate of $O(k^{-1/3})$. This gap between impressive practical performance and poor theoretical guarantees was an open question for a long period of time. In this paper, we make a significant step to close this gap. We improve upon the existing rate and propose the Cubic Regularized Quasi-Newton Method with a convergence rate of $O(k^{-1})$. The key to achieving this improvement is to use the Cubic Regularized Newton Method over the Damped Newton Method as an outer method, where the Quasi-Newton update is an inexact Hessian approximation. Using this approach, we propose the first Accelerated Quasi-Newton method with a global convergence rate of $O(k^{-2})$ for general convex functions. In special cases where we can improve the precision of the approximation, we achieve a global convergence rate of $O(k^{-3})$, which is faster than any first-order method. To make these methods practical, we introduce the Adaptive Inexact Cubic Regularized Newton Method and its accelerated version, which provide real-time control of the approximation error. We show that the proposed methods have impressive practical performance and outperform both first and second-order methods.

On bounded oscillation operators

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Bounded oscillation (BO) operators were recently introduced and studied in [2]. Those are operators running on abstract measure spaces equipped with a ball-basis. It was proved that many operators in harmonic analysis (Calderón-Zygmund operators and their maximally modulations, Carleson type operators, martingale transforms, Littlewood-Paley square functions, maximal function, etc) are BO operators. Various properties of BO operators were studied in [3, 2]. A multilinear version of BO operators recently were investigated in [1], recovering many results of papers [3, 2] in multilinear setting. One of the main lines of these investigations are sparse dominations results, a subject of intensive study of recent years. First sparse domination results were proved for the Calderón-Zygmund operators in [6, 7, 5]. The subject of sparse domination is closely related to weighted norm and exponential decay estimates of various operators.

We will consider a generalized version of BO operators, involving new parameters in their definition, as well as considering the operators on vector-valued function spaces. With this definition we capture more operators to the class of BO operators. For a class of BO operators we obtain new type of sparse estimation, involving mean oscillation instead of integral averages in the definition of sparse operators. Among with new corollaries we recover also series of results obtained in recent years. In particular, we prove the boundedness of maximally modulated Calderón-Zygmund operators on spaces BMO. We provide a new simplified approach, separating a certain set theoretic proposition, which became a basic tool in the proofs of the main results.

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On weighted integral representations of holomorphic functions in tube domains over real Siegel domains

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Denote by $G = \{y = (y_1, y') \in R^n : y_1 \in R, y' \in R^{n-1}, y_1 > |y'|^2\}$ so-called real Siegel domain in the space R^n and let $T_G = \{x + iy \in C^n : x \in R^n, y \in G\}$ be the tube domain over G. Assume also that a positive continuous function $\varphi(t), t \in (0; +\infty)$, satisfies the condition

$$\liminf_{t \to +\infty} \frac{ln\varphi(t)}{t} \geq 0. \tag{1}$$

Finally, suppose that $1 \leq p \leq 2$ and the constants s, r, σ are positive.

Denote by $H^{p,s}_{\varphi,r,\sigma}(T_G)$ the set of all functions $f(z) \equiv f(x + iy)$ holomorphic in the tube domain T_G and satisfying the condition

$$M^{p,s}_{\varphi,r,\sigma}(f) \equiv \int_G \left(\int_{R^n} |f(x+iy)|^p dx \right)^s \cdot \\ \cdot \varphi(y_1 - |y'|^2) \cdot e^{-ry_1} \cdot e^{-\sigma|y'|^2} dy < +\infty. \tag{2}$$

The following Paley-Wiener type theorem is valid:

Theorem. *For each function $f \in H^{p,s}_{\varphi,r,\sigma}(T_G)$ there exists a measurable function $F(t_1, t'), t_1 > -\frac{r}{s}, t' \in R^{n-1}$ (satisfying certain growth condition in terms of $M^{p,s}_{\varphi,r,\sigma}(f)$) such that f admits an integral representation of the form*

$$f(z) = \frac{1}{(2\pi)^{n/2}} \int_{-\frac{r}{s}}^{+\infty} \int_{R^{n-1}} F(t_1, t') e^{iz_1 t_1 + i<z', t'>} dt_1 dt', \;\; (z_1, z') \in T_G. \tag{3}$$

Based on these representations (and under additional conditions on the weight function φ) it will be possible to construct reproducing kernels for the classes $f \in H^{p,s}_{\varphi,r,\sigma}(T_G)$.

On number fields defined by $x^5 + ax + b$

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Let $K = \mathbb{Q}(\theta)$ be an algebraic number field with θ a root of an irreducible quintic trinomial of the type $x^5 + ax + b \in \mathbb{Z}[x]$. If ind θ denotes the index of the subgroup $\mathbb{Z}[\theta]$ in A_K and $i(K)$ stand for the index of the field K defined by

$$i(K) = \gcd\{\text{ind } \alpha \mid K = \mathbb{Q}(\alpha) \text{ and } \alpha \in A_K\}.$$

A prime number p dividing $i(K)$ is called a prime common index divisor of K. In this talk, for every rational prime p, we provide necessary and sufficient conditions on a, b so that p is a common index divisor of K. In particular, we give sufficient conditions on a, b for which K is non-monogenic. Also, we compute the highest power of each prime p dividing the discriminant of K besides explicitly constructing an integral basis of K. In the end, I will illustrate these results through examples.

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Controllable Video Generation with Diffusion Models

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Recent text-to-video generation approaches rely on computationally heavy training and require large-scale video datasets. In this paper, we introduce a new task of zero-shot text-to-video generation and propose a low-cost approach (without any training or optimization) by leveraging the power of existing text-to-image synthesis methods (Stable Diffusion), making them suitable for the video domain. Our key modifications include (i) enriching the latent codes of the generated frames with motion dynamics to keep the global scene and the background time consistent; and (ii) reprogramming frame-level self-attention using a new cross-frame attention of each frame on the first frame, to preserve the context, appearance, and identity of the foreground object. Experiments show that this leads to low overhead, yet high-quality and remarkably consistent video generation. Moreover, our approach is not limited to text-to-video synthesis but is also applicable to other tasks such as conditional and content-specialized video generation, and Video Instruct-Pix2Pix, instruction-guided video editing. As experiments show, our method performs comparably or sometimes better than recent approaches, despite not being trained on additional video data.

Duality of Energy and Probability in Models of Statistical Physics

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The entire theoretical component of statistical physics is based on the fundamental Gibbs (Boltzmann-Gibbs) formula, which establishes a relationship between the probability of finding a physical system in a certain state with the potential energy (Hamiltonian) of this state.

Much attention has been paid to the issue of justification of the Gibbs formula, usually using physical reasoning. At the same time, such an impressive theory (statistical physics) has been built based on this formula that the Gibbs formula can be accepted as a postulate.

The rigorous (mathematical) justification of the Gibbs formula for infinite systems was proposed in [1] based on the notion of the transition energy field. An analogous approach was applied in the case of finite systems in [2].

In this talk, we will show that there is a much deeper relationship between probability and energy than that reflected in the classical version of the Gibbs formula. Namely, we will show that energy and probability are dual concepts. Some applications of this fact will also be discussed.

The talk is based on the joint works with Boris S. Nahapetian (Institute of Mathematics, NAS RA).

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On continuous and Lipschitz selections of multivalued mappings

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The article considers a multivalued mapping of the form

$$a(x) = \{y \in Y / f_i(x,y) \leq 0 \;\; i \in I\},$$

where $x \in R^m$, Y is closed convex subset of the space R^n, I is finite set of indexes and gradients $f'_{iy}(x,y)$, $i \in I$(with respect to variable y) of functions $f_i(x,y)$, $i \in I$ satisfy the Lipshitz condition. Using linearization method, we obtain theorems on the existence of continuous and Lipshitz selections passing through any point on the graph of the mapping a.

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On correct solvability of dirichlet problem in a half-space for regular equations with non-homogeneous boundary conditions

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In this paper we consider the following Dirichlet problem with non homogeneous boundary conditions in a multianisotropic Sobolev space $W_2^{\mathfrak{M}}(R^2 \times R_+)$.

$$\begin{cases} P(D_x, D_{x_3})u = f(x, x_3), \ x_3 > 0, \ x \in R^2, \\ D_{x_3}^s u\big|_{x_3=0} = \varphi_s(x), \ s = 0, \cdots, m-1. \end{cases}$$

It is assumed that $P(D_x, D_{x_3})$ is a multianisotopic regular operator of a special form with characteristic polyhedron $\mathfrak{M}$. In paper [1] a similar problem is considered with homogeneous boundary conditions in space $W_p^{\mathfrak{M}}(R^{n-1} \times R_+)$. Using a special integral representation, containing all generalized derivatives of a function, corresponding to the vertices of the polyhedron $\mathfrak{M}$ (see [2]), in [1] an approximate solution for the problem with homogeneous boundary conditions was constructed and conditions for its unique solvability were obtained. In this paper, using the results from [3], related to the traces of functions from multianisotropic Sobolev spaces, unique solvability conditions are obtained for the problem with inhomogeneous boundary conditions.

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Convergence of remote projections onto convex sets

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We try to find a point in the intersection of closed convex sets by iterating the nearest point projection onto them. This works, if the sets are symmetric and we always project on the most distant set. We will discuss to what extent this assumptions can be dropped.

Let $\{C_\lambda\}_{\lambda\in\Lambda}$ be a family of closed and convex sets in a Hilbert space H. Assume the sets C_λ are also symmetric and $\{x_n\}_{n\in\mathbb{N}}$ is a sequence of remote projections onto them. This means, $x_0 \in H$, and x_{n+1} is the projection of x_n onto the most distant set C_λ. Then the sequence $\{x_n\}$ converges to a point in the intersection $\bigcap_{\lambda\in\Lambda} C_\lambda$. We give examples explaining to what extent the symmetry condition on the sets C_λ can be dropped.

Joint work with P. Borodin.

Irreducibility of random polynomials

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Take a random polynomial with random, independent +/-1 coefficients. What is the probability that it is irreducible over the rationals, asymptotically as the degree goes to infinity? This captivating problem, still open, has seen a lot of progress in recent years. We will survey this recent progress, including our recent result with Lior Bary-Soroker and Dimitris Koukoulopoulos.

Predictive algorithms for external force identification

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In this talk, I will describe how some ideas of dynamical sampling can be used to design algorithms for estimating bursts in the external force of an abstract initial value problem (IVP). The design yields simple algorithms for quick and accurate estimation of Dirac or rapidly decaying bursts. The predictive nature of the algorithms manifests in using the current samples to estimate the measurements at the end of the following sampling period on condition that no burst occurred within the period. The talk is based on joint work with A. Aldroubi, L. Gong, L. Huang, K. Kornelson, and B. Miller.

Hidden Markov Processes and Adaptive Filters

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The problems of parameter estimation and adaptive filters construction are considered for several partially observed linear models including hidden O-U process (ergodic and with small noise), AR process and Telegraph process. For each model on-line estimators of the unknown parameters are proposed and then these estimators are used for the construction of adaptive filters. The questions of optimality of these filters are discussed as well .

Approximation by refinement masks

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We study density of the set of all refinement masks for framelets. For an arbitrary 2π-periodic function f such that $f(0) = 1$ and $|f(x)|^2 + |f(x + \pi)|^2 \leq 1$ we design a Parseval wavelet frame with a compact support. The corresponding refinement mask uniformly approximates f. The corresponding refinable function has stable integer shifts. The assumption on f is natural, since to design wavelet frame we exploit the unitary extension principle and any refinement mask m_0 has to satisfy the same inequality $|m_0(x)|^2 + |m_0(x + \pi)|^2 \leq 1$.

Solving boundary value problems with moving boundaries using the method of constructing solutions integro-differential equations

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The problem of vibrations of objects with moving boundaries, formulated as a differential equation with boundary and initial conditions, is a nonclassical generalization of a hyperbolic problem. To facilitate the construction of the solution to this problem and to justify the choice of the form of the solution, equivalent integro-differential equations with symmetric and time-dependent kernels and time-varying integration limits are constructed. The phenomenon of steady-state resonance and passage through resonance is investigated using numerical methods. The solution was made in the Matlab environment of dimensionless variables, which allows one to use the obtained results to calculate oscillations of a wide range of technical objects Among all the many problems of the dynamics of elastic systems from the point of view of technical applications, the problems of oscillations in systems with time–varying geometric dimensions are very relevant. Systems, the boundaries of which are moving, are widespread in technology (ropes of hoisting installations [1–3], flexible transmission links [3], solid fuel rods [4,5], drill strings [5], etc.). The studies of many authors on the dynamics of lifting ropes have led to the need to formulate new problems in mechanics concerning the dynamics of one–dimensional objects of variable lengths. The presence of moving boundaries causes significant difficulties in the description of such systems; therefore, approximate solution methods are mainly used here [1–5].

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GradSkip: Communication-Accelerated Local Gradient Methods with Better Computational Complexity

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In this work, we study distributed optimization algorithms that reduce the high communication costs of synchronization by allowing clients to perform multiple local gradient steps in each communication round. Recently, Mishchenko et al. (2022) proposed a new type of local method, called ProxSkip, that enjoys an accelerated communication complexity without any data similarity condition. However, their method requires all clients to call local gradient oracles with the same frequency. Because of statistical heterogeneity, we argue that clients with well-conditioned local problems should compute their local gradients less frequently than clients with ill-conditioned local problems. Our first contribution is the extension of the original ProxSkip method to the setup where clients are allowed to perform a different number of local gradient steps in each communication round. We prove that our modified method, GradSkip, still converges linearly, has the same accelerated communication complexity, and the required frequency for local gradient computations is proportional to the local condition number. Next, we generalize our method by extending the randomness of probabilistic alternations to arbitrary unbiased compression operators and considering a generic proximable regularizer. This generalization, GradSkip+, recovers several related methods in the literature. Finally, we present an empirical study to confirm our theoretical claims.

Covariogram of a Quadrilateral Prism

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We aim to establish a formula for a covariogram of an arbitrary right convex quadrilateral prism through orientation-dependent characteristics of its base D. In the Cartesian plane, we replace D by its *standard image*, $D_s = [b, \alpha_0, \alpha, \beta, \beta_0]$, where $b > 0$, $0 < \alpha_0 < \alpha \leq \beta < \beta_0 < \pi$, and $\alpha \leq \pi/2$. D_s is a quadrilateral, congruent to D, which holds a horizontal "base" of length b, and is determined by the inclination angles of the "legs" (α and β) and diagonals (α_0 and β_0).

Five orientation-dependent characteristics of D_s are introduced and explicitly evaluated for every direction $\phi = (\cos\varphi, \sin\varphi) \in \mathbb{S}^1$. Two of them, $x_1(\varphi)$ and $x_0(\varphi)$, stand for the lengths of the *first and the second-order φ-diameters* of D_s (we generalized the concept of a φ-diameter of a polygon introduced in [1]). The other three, $\ell_0(\varphi), \ell(\varphi)$, and $\ell_1(\varphi)$, which we call *supplementary measures* of D_s, are properly chosen to satisfy the identity $b_{D_s}(\varphi) = \ell_0(\varphi) + \ell(\varphi) + \ell_1(\varphi)$, where $b_{D_s}(\varphi)$ is the L_1-measure of the orthogonal projection of D_s onto the orthogonal complement of the subspace spanned by ϕ.

A simple formula for the orientation-dependent chord length distribution function of D_s in terms of the mentioned characteristics is found. Then, applying Matheron's [2] technique, the corresponding formula for the covariogram of D_s is also established. For more general result, let D_s^h be the right prism $\{(x, y, z) : (x, y) \in D_s, 0 < z \leq h\}$, where D_s is a standard image of a convex quadrilateral. For a vector $\omega = (\cos\varphi\cos\theta, \sin\varphi\cos\theta, \sin\theta) \in \mathbb{S}^2$, let $\omega^\perp$ be the orthogonal complement of $\{t\omega : t \in \mathbb{R}\}$ in $\mathbb{R}^3$, and $\Pi_{D_s^h}(\varphi, \theta)$ be the orthogonal projection of D_s^h onto the plane $\omega^\perp$. Let $l_{(\varphi,\theta)} + y$ be the line that passes through $y \in \omega^\perp$ and has direction vector ω. We denote $x_{\max}(\varphi, \theta) = \max\limits_{y \in \Pi_{D_s^h}(\varphi,\theta)} L_1\big((l_{(\varphi,\theta)} + y) \cap D_s^h\big)$.

Theorem 1. *For a $\varphi \in [0, \pi)$, let x_1 and x_0 be the lengths of the first and the second-order φ-diameters of D_s, respectively. Let ℓ_0, ℓ, ℓ_1 be the supplementary φ-measures of D_s. Then, for the direction $\omega = (\cos\varphi\cos\theta, \sin\varphi\cos\theta, \sin\theta)$, $0 \leq \theta \leq \frac{\pi}{2}$, the covariogram $C_{D_s^h}(t\omega)$ has the following representation:*

(a) *If $\tan^{-1}\frac{h}{x_0} < \theta \leq \frac{\pi}{2}$ and $0 \leq t < x_{\max}(\varphi, \theta)$, or $0 \leq \theta \leq \tan^{-1}\frac{h}{x_0}$ and $0 \leq t < x_0 \sec\theta$, then*

$$C_{D_s^h}(t\omega) = \left(\|D_s\| - b_{D_s}\cos\theta \cdot t + \frac{1}{2}\left(\frac{\ell_0}{x_0} + \frac{\ell_1}{x_1}\right)\cos^2\theta \cdot t^2\right)(h - \sin\theta \cdot t);$$

(b) *If $0 \leq \theta \leq \tan^{-1}\frac{h}{x_0}$ and $x_0 \sec\theta \leq t < x_{\max}(\varphi, \theta)$, then $x_0 < x_1$ and*

$$C_{D_s^h}(t\omega) = \frac{1}{2}\left(\frac{\ell}{x_1 - x_0} + \frac{\ell_1}{x_1}\right)(x_1 - \cos\theta \cdot t)^2(h - \sin\theta \cdot t).$$

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Melikyan Algebras in Classification of Simple Modular Lie Algebras

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The problem of classifying finite-dimensional simple Lie algebras over fields of positive characteristics $p > 0$ is a long standing one.The same problem for finite-dimensional Lie algebras over an algebraically closed field of characteristic zero, was almost achieved in the works of W. Killing and E. Cartan during the last decade of XIX century. A decade later, after the classification of finite-dimensional simple Lie algebras, E. Cartan classified the simple infinite-dimensional Lie algebras of complex vector fields on a finite-dimensional space. Nowadays, these algebras are called the simple Cartan Lie algebras, and they are: the general Lie algebra W_n, the special Lie algebra S_n, the Hamiltonian Lie algebra H_n and the contact Lie algebra K_n.

Work on simple Lie algebras of prime characteristic began almost 90 years ago. Around the year 1937 E. Witt came up with an example of a simple Lie algebra of dimension p, which behaved completely differently from the known Lie algebras of characteristic zero. Witt algebra was generalized by N. Jacobson, and same time he introduced notion of restricted Lie algebras (also called Lie p-algebras). Over the thirty years following the discovery of Witt, several new families of simple modular Lie algebras were found and studied. In 1966 A. Kostrikin and I. Shafarevich introduced four families of simple finite-dimensional restricted Lie algebras that covered all known simple non-classical Lie algebras. Thus algebras were the finite dimensional analogs of the infinite simple Lie algebras of Cartan, over the field of nonzero characteristic. They called these algebras Cartan type Lie algebras. Same time they conjectured that over an algebraically closed field of characteristic $p > 5$ a finite dimensional restricted simple Lie algebra is classical or Cartan type.

In 1988 R. Block and R. Wilson proved that a finite dimensional restricted simple Lie algebra over the algebraically closed field of characteristic $p > 7$ is classical or Cartan type, which in part conforms the Kostrikin-Shafarevich conjecture. The Block-Wilson classification marked a major breakthrough in the theory and, also provided a framework for the classification of the nonrestricted simple Lie algebras. Finally, two decades after the Block-Wilson classification, A. Premet and H. Strade not only conformed the Kostrikin-Shafarevich original conjecture also completed classification simple modular Lie algebras over an algebraically closed field of characteristic $p > 3$.

The final Block-Wilson-Strade-Premet Classification Theorem, that is considered as landmark result of modern mathematics, states:

Every finite-dimensional simple Lie algebra over an algebraically close field of characteristic $p > 3$ is classical, Cartan, or Melikyan type.

The aim of this talk is to give a brief introduction to classical, Cartan, and Melikyan algebras, then discuss recent results and open problems.

Various problems arising when restoring a partially filled matrix of distances between DNA chains

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We continue to consider one of the tasks of biocybernetics, i.e. the problem of reconstructing the distance matrix between DNA sequences. In this problem, not all the elements of the matrix under consideration are known at the input of the algorithm (usually 50% or less of the elements). The basis for the development of the algorithm for reconstructing such a matrix is the method of comparative evaluation of algorithms for calculating distances between DNA sequences developed and investigated by us, based on a special analysis of the distance matrix. In this analysis, we applied the badness of each of the triangles of the matrix determined by us before. Continuing to improve the algorithms for solving this problem, we consider the use of the branches and bounds method in it. To do this, for some known sequence of unfilled elements, we apply the algorithms we considered in previous publications, but now we choose the sequences ourselves using developed by us version of method of branches and bounds.

In our interpretation of this method, all possible sequences of unknown elements of the upper triangular part of the matrix are taken as the set of admissible solutions. In each current subtask, any of the blank elements of the matrix is taken as the separating element, and the sum of the badness values for all triangles that have already been formed by the time this subtask is considered is taken as the boundary. Thus, the definition of elements of an incompletely filled matrix occurs in such a sequence that the final badness indicator for all triangles is selected using greedy heuristics that fits completely into the framework of the classical variants of the description of the branches and bounds method.

As a result of applying such an algorithm, we get results that, from the point of view of the value of badness, are the lowest possible (in the case of a completed version of the branch and boundary method), or close to optimal ones (in the case of its uncompleted version). At the same time, in our computational experiments, the running time of the algorithm practically coincides with the time of the algorithm considered by us in the previous publication (it exceeds it by no more than 10%), and the badness value usually decreases by 10-20% from the initial value. Thus, we are able to quickly and efficiently restore the DNA matrix, often even if it is filled less than 20%.

A study of the union of semilattices on the set of subsets of grids of Waterloo language

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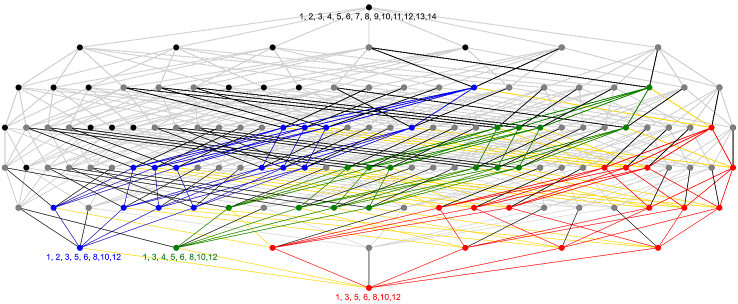
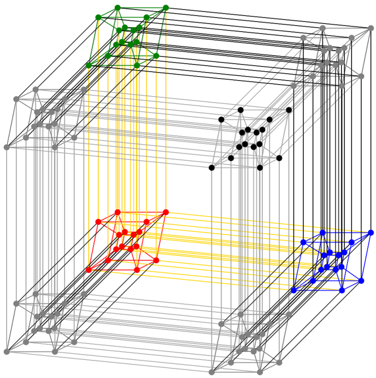
We continue to consider the set of grids of Waterloo language. The set of all covering sets of grids (and, respectively, the set of all covering automata) forms a semilattice by union (since the union of two covering sets is a covering set). However, the set of all covering sets of grids does not form a semilattice on intersection. It can be said that the set of all covering sets of grids is a union of intersection semilattices.



As a result of the algorithm, we obtain the set of semilattices of covering automata, which will reveal additional regularities associated with the Waterloo automaton:



Embeddings of groups and the universal elements in free groups

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Our objective is to presentin which we construct an algorithm for explicit embedding of any countable group G, given by its generators and defining relations, into such a 2-generator group T that the generators and defining relations of T are automatically deduced from those of G.

By the famous theorem of Higman, Neumann and Neumann any countable group G is embeddable into a 2-generated group T [1]. The original embedding method of [1] already is explicit, but we need a method that makes discovery of the relations of T a simple, automated task, and also preserves certain features in relations required for embeddings of recursive groups into finitely presented groups, which is possible by the well-known theorem of Higman [2], see also embedding properties in [3], [4], [5].

In the free group $F_2 = \langle x, y \rangle$ of rank 2 single out the special *universal words* $a_i(x,y) = y^{(xy^i)^2 x^{-1}} y^{-x}$, $i = 1, 2, \ldots$ Assume a countable group G is given by its generators and defining relations as $G = \langle\, A \mid R \,\rangle = \langle a_1, a_2, \ldots \mid r_1, r_2, \ldots \rangle$ where the m'th relation $r_m = r_m(a_{i_{m,1}}, \ldots, a_{i_{m,k_m}}) \in R$ is a word of length k_m on certain letters $a_{i_{m,1}}, \ldots, a_{i_{m,k_m}} \in A$. If we replace in r_m each of $a_{i_{m,j}}$, $j = 1, \ldots, k_m$, by the respective word $a_{i_{m,j}}(x,y)$ given above, then we get a new word $r'_m(x,y) = r_m\big(a_{i_{m,1}}(x,y), \ldots, a_{i_{m,k_m}}(x,y)\big) \in F_2$ relying on just two letters x and y. In these notations the main embedding of [6] is given by:

Theorem 1. *For any countable group $G = \langle a_1, a_2, \ldots \mid r_1, r_2, \ldots \rangle$ the map $\gamma : a_i \to a_i(x,y)$, $i = 1, 2, \ldots$, defines an injective embedding of G into the 2-generator group*

$$T_G = \big\langle x, y \;\mid\; r'_1(x,y),\; r'_2(x,y), \ldots \,\big\rangle$$

given by its relations $r'_m(x,y)$, $m = 1, 2, \ldots$

When G is a *torsion-free* group, then the above mentioned words $a_i(x,y)$ can be replaced by somewhat shorter alternatives: $\bar{a}_i(x,y) = a^{t_i} = y^{(xy^i)^2 x^{-1}}$, $i = 1, 2, \ldots$

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New monotonicity formulas for the curve shortening flow in $\mathbb{R}^3$

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The curve shortening problem is one of the most beautiful and classical problems in geometric PDEs, which models a curve moving by its curvature vector.

Let $\tilde{\gamma} : S^1 \times \mathbb{R}_+ \to \mathbb{R}^3$ be the rescaled curve shortening flow in 3D, satisfying

$$\partial_\tau \tilde{\gamma} = \frac{1}{2}\tilde{\gamma} + \frac{\tilde{\gamma}' \times (\tilde{\gamma}'' \times \tilde{\gamma}')}{|\tilde{\gamma}'|^4}, \tag{1}$$

where the rescaled space and time parametrization runs on $(x,\tau) \in S^1 \times \mathbb{R}_+$, with S^1 the unit circle, and $'$ means the derivative in $x \in S^1$ variable. Further let

$$\psi = \arcsin \frac{\langle \tilde{\gamma}, \tilde{\gamma}' \rangle}{|\tilde{\gamma}||\tilde{\gamma}'|} \in [-\tfrac{\pi}{2}, \tfrac{\pi}{2}]$$

be the angle between the position vector $\tilde{\gamma}$ and the plane orthogonal to the tangent vector $\tilde{\gamma}'$, with a sign coming from the sign of $\langle \tilde{\gamma}, \tilde{\gamma}' \rangle$,

$$\kappa = \frac{|\tilde{\gamma}'' \times \tilde{\gamma}'|}{|\tilde{\gamma}'|^3} \tag{2}$$

be the curvature, and

$$\nu = \frac{\tilde{\gamma}' \times (\tilde{\gamma}'' \times \tilde{\gamma}')}{|\tilde{\gamma}'||\tilde{\gamma}'' \times \tilde{\gamma}'|}$$

be the normal vector.

We prove three new monotonicity formulas. The fist monotonicity formula deals with the length of the projection of the curve on the unit sphere in 3D, and is formulated in the theorem below.

Theorem 1. *Let $\tilde{\gamma}$ be the rescaled curve shortening flow in (1). Then*

$$\frac{d}{d\tau} \int_{S^1} \frac{|\tilde{\gamma}'|}{|\tilde{\gamma}|} \cos \psi dx = - \int_{S^1} \frac{|\tilde{\gamma}'|}{|\tilde{\gamma}|^3 \cos^3 \psi} \kappa^2 \left|\mathrm{Proj}_{\nu \times \tilde{\gamma}'} \tilde{\gamma}\right|^2 dx - 2 \sum_{\psi(x) = \pm \frac{\pi}{2}} \frac{\kappa(x)}{|\tilde{\gamma}(x)|}, \tag{3}$$

where the second sum is taken over the points where $\psi = \pm \frac{\pi}{2}$ (or $\tilde{\gamma} \parallel \tilde{\gamma}'$). Further, let $a \in C([0,\infty))$ be an arbitrary continuous function on $[0,\infty)$. Then

$$\int_{S^1} a(|\tilde{\gamma}|)|\tilde{\gamma}'| \sin \psi dx = 0. \tag{4}$$

The second monotonicity formula we prove is the generalization of the classical formula of G. Huisken, while the third one is the generalization of the monotonicity formula with a logarithmic term, previously derived by the author for planar curves. See [1] for details and further references.

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Multiple power series continuability into a sectorial domain

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One way to explore on analytic function is to expand it to a power series. The coefficients of a power series expansion contain the information of its analytic continuation. One possible approach to treat analytic continuation problem is to interpolate the coefficients.

In case of a one variable power series, Arakelian gave a criterion for a given arc of a unit circle to be an arc of regularity in terms of the indicator of the interpolating entire function. Pólya found conditions for analytic continuability of a series to the whole complex plane except for some boundary arc. Le Roy and Lindelof obtained conditions under which the series is analytically extends in to the sectoral domain.

We give conditions for analytic extension of a multiple power series to a sectorial domain. Consider the multiple power series

$$f(z) = \sum_{k \in \mathbb{N}^n} f_k z^k. \tag{1}$$

Following V. Ivanov we introduce the set which implicitly contains the notion of the growth indicator of an entire function $\varphi(z) \in \mathcal{O}(\mathbb{C}^n)$:

$$T_\varphi(\theta) = \{\nu \in \mathbb{R}^n : \ln|\varphi(re^{i\theta})| \le \nu_1 r_1 + ... + \nu_n r_n + C_{\nu,\theta}\},$$

where the inequality is satisfied for any $r \in \mathbb{R}^n_+$ with some constant $C_{\nu.\theta}$.

Denote

$$T_\varphi := \bigcap_{\theta_j = \pm\frac{\pi}{2}} T_\varphi(\theta_1, ..., \theta_n),$$

$$\mathcal{M}_\varphi := \{\nu \in [0, \pi)^n : \ \nu + \varepsilon \in T_\varphi, \ \ \nu - \varepsilon \notin T_\varphi \ \ \text{for any} \ \ \varepsilon \in \mathbb{R}^n_+\}.$$

Let G be a sectorial set

$$G = \bigcup_{\nu \in \mathcal{M}_\varphi} \{z \in \mathbb{C}^n : \nu_j < \arg z_j < 2\pi - \nu_j, \ j = 1, ..., n, \}. \qquad (2)$$

Theorem. *Let $\varphi(\zeta)$ be an entire function of the exponential type interpolating coefficients f_k of the series (1). If there is $\nu(\theta) \in \mathcal{M}_\varphi(\theta)$ for $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]^n$ that satisfying the inequalities*

$$\nu_j(\theta) \leq a|\sin\theta_j| + b\cos\theta_j, \ \ j = 1, ..., n,$$

where $a \in [0, \pi)$, $b \in [0, \infty)$.

Then the sum of the series extends analytically to a sectorial domain G of the form (2).

Nonparametric Estimation in Some Statistical Inverse Problems

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The problem of recovering unknown functions (distributions) from incomplete information (the moments) will be considered. Under mild conditions on the underlying functions, the rate of approximations for proposed constructions are discussed. Several applications of proposed constructions in statistical inverse problems, e.g., deconvolution, multiplicative-censoring models, mixtures, and image reconstruction from moments will be suggested. New estimates of conditional distribution and corresponding conditional quantile function, as well as the shapes of unknown sets based on the sequence of empirical moments will be introduced. Comparisons of theoretical and approximated curves will be illustrated via simulation studies.

Some quantitative estimates for the Carleson operator of the scattering transform

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The scattering transform, also called the non-linear Fourier transform, is an object that comes up in many fields of analysis and PDEs, such as Korteweg de Vries equation, Schrödinger equation, orthogonal polynomials on the unit circle etc. The question of its almost everywhere convergence, a non-linear analog of the celebrated Carleson theorem [1], came up in connection to the spectral theory of one-dimensional Schrödinger operators. For potentials in L^p with $1 \leq p < 2$, the almost everywhere convergence, together with the appropriate maximal Hausdorff-Young bounds for the Carleson operator follows from the work of Christ and Kiselev [2], as summed up by Tao and Thiele [3]. Christ and Kiselev's method relies on bounds of multilinear expansion of the scattering transform. However, this expansion fails in L^2 as shown by Muscalu, Tao and Thiele [7].

Recently, Alexei Poltoratski proved [4] the pointwise convergence of the non-linear Fourier transform giving a partial answer to the long-standing question of Muscalu, Tao and Thiele [5]. However, unlike in the linear setup where Stein's principle [6] applies, Poltoratski's result does not imply the L^2 bound for the maximal function. His method relies on detailed analysis of zeros of so called de Branges entire functions through

Ricatti equations. We quantify the techniques of Poltoratski and, in particular, give a new proof of a weaker version of maximal Hausdorff-Young estimate. Meanwhile, we obtain quantitative estimates for the de Branges function associated to the non-linear Fourier transform through its zeros and the Hardy-Littlewood maximal function of the spectral measure. As a corollary to these estimates, we obtain a zero free strip of the de Brange function for potentials with small L^1 norm.

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On linearity of universal algebras with second order formulas

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In this work the Belousov theorem on linearity of invertible algebras with the Schauffler $\forall\exists(\forall)$-identity is extended to regular division algebras for other $\forall\exists(\forall)$-identities. For the formulas at consideration, the Schauffler-type theorems are also proved. The results are applicable in cryptography (cf. [1, 2, 3]).

Definition 1. *A regular division binary algebra $(Q;\Sigma)$ is called r-algebra, if there exists invertible operation $A \in \Sigma$.*

Theorem 1. *r-algebra $(Q;\Sigma)$ with the one of the following $\forall\exists(\forall)$-identities of associativity:*

$$\forall X, Y \exists X', Y' \forall x, y, z X(Y(x,y),z) = X'(x, Y'(y,z)), \tag{1}$$

$$\forall X, Y \exists X', Y' \forall x, y, z X(x, Y(y,z)) = X'(Y'(x,y),z), \tag{2}$$

$$\forall X, Y \exists X', Y' \forall x, y, z X(Y'(x,y),z) = Y(x, X'(y,z)), \tag{3}$$

$$\forall X, Y \exists X', Y' \forall x, y, z X(x, X'(y,z)) = Y'(Y(x,y),z), \tag{4}$$

$$\forall X, Y \exists X', Y' \forall x, y, z Y'(x, X(y,z)) = Y(X'(x,y),z), \tag{5}$$

is endo-linear on a group.

Theorem 2. *Let $(Q;\Omega)$ be algebra of all regular division operations. One of the* (1), (2), (3), (4), (5) *$\forall\exists(\forall)$-identities of associativity holds in $(Q;\Omega)$, if and only if $|Q| \leq 3$.*

Theorem 3. *r-algebra $(Q;\Sigma)$ with the one of the following $\forall\exists(\forall)$-identities of mediality and paramediality:*

$$\forall X,Y\exists X',Y'\forall x,y,u,vX(Y(x,y),Y(u,v)) = X'(Y'(x,u),Y'(y,v)), \quad (6)$$

$$\forall X,Y\exists X',Y'\forall x,y,u,vX'(Y'(x,y),Y'(u,v)) = X(Y(x,u),Y(y,v)), \quad (7)$$

$$\forall X,Y\exists X',Y'\forall x,y,u,vX(Y(x,y),Y(u,v)) = X'(Y'(v,y),Y'(u,x)), \quad (8)$$

$$\forall X,Y\exists X',Y'\forall x,y,u,vX'(Y'(x,y),Y'(u,v)) = X(Y(v,y),Y(u,x)), \quad (9)$$

$$\forall X,Y\exists X',Y'\forall x,y,u,vX(Y'(x,y),Y'(u,v)) = Y(X'(x,u),X'(y,v)), \quad (10)$$

$$\forall X,Y\exists X',Y'\forall x,y,u,vX(Y'(x,y),Y'(u,v)) = Y(X'(v,y),X'(u,x)), \quad (11)$$

is endo-linear on a group.

Theorem 4. *Let $(Q;\Omega)$ be algebra of all regular division operations. One of the (6), (7), (8), (9), (10), (11) $\forall\exists(\forall)$-identities of mediality or paramediality holds in $(Q;\Omega)$, if and only if $|Q| \leq 3$.*

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Approximate controllability of PDEs using only a few Fourier modes

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The aim of this talk is to provide a brief introduction to the Agrachev-Sarychev approach for controllability of partial differential equations (PDEs). The focus will be on PDEs with polynomial nonlinearity and control force acting directly only on a small number of Fourier modes. We will present a necessary and sufficient condition for global approximate controllability.

Reconstruction of bodies in $\mathbf{R}^n$ using tomographic methods of stochastic geometry

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The present report contains a review of the main results of Yerevan research group in tomography of bounded convex bodies. Let $\mathbf{R}^n$ $(n \geq 2)$ be the n-dimensional Euclidean space, $\mathbf{D} \subset \mathbf{R}^n$ be a bounded convex body. Random k-flats in $\mathbf{R}^n$, $1 \leq k \leq n-1$ generate cross sections of random size in convex body $\mathbf{D}$. As $\mathbf{D}$ is a convex body, then obviously intersections of k-flats with $\mathbf{D}$ are always connected subsets of $\mathbf{R}^n$ for every $k \in \{1, ..., n-1\}$. The determination of the distribution of size of cross sections has a long tradition of application to collections of bounded convex bodies forming structures in metal and crystallography. However, calculations of geometrical characteristics of random cross sections is often a difficult task. In a special case $k = 1$ we call the corresponding distribution function as the chord length distribution function. For $n = 2$ the list of known results was expanded after 2005 when N. G. Aharonyan and V. K. Ohanyan obtained the explicit formula of the chord length distribution function for a regular polygon (see [1]). A computer program is created which gives values of a chord length distribution function in the case of a regular n-gon for every natural $n \geq 3$ (see [3]). A practical application these results in crystallography can be found in [2] and [4]. These all problems have applications in Medicine Tomography.

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Nonparametric Statistics for SPDEs

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Stochastic partial differential equations (SPDEs) are used more and more often to model real-world phenomena. Currently, statistical methodology for these equations driven by space-time white noise is developing rapidly. Based on the classical spectral method for parametric drift estimation, we shall exhibit fundamental differences with the case of stochastic ordinary differential equations. This method, however, is restricted to simple parametric situations and we discuss the local estimation method in detail, which allows to estimate varying coefficients in the differential operator of a parabolic SPDE nonparametrically with optimal rates. This approach is extended to observations under measurement errors ('static noise'), showing a fundamentally different impact of dynamic and static noise levels. Finally, we present an abstract minimax lower bound framework for stochastic evolution equations generated by normal operators in Hilbert space and obtain a rich picture of complexity for different SPDE estimation objectives. Some illustrations with cell motility experiments in biophysics are provided.

On the 5th generation of local training methods in federated learning

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I will outline the history of the theoretical development of the local training "trick" employed in virtually all successful federated learning algorithms. In particular, I will identify five distinct generations of meth-

ods and results: 1) heuristic, 2) homogeneous, 3) sublinear, 4) linear and 5) accelerated. The 5th generation, initiated by the ProxSkip algorithm by Mishchenko et al (ICML 2022), finally led to the proof that local training, if carefully executed, leads to provable acceleration of communication complexity, without requiring any data homogeneity assumptions. Because these latest advances are very new, there are many opportunities to develop the 5th generation of local training methods further. I will give a brief overview of what we know now, and what problems still remain open.

5th generation local training methods known at the moment:

ProxSkip: "ProxSkip: Yes! Local Gradient Steps Provably Lead to Communication Acceleration! Finally!"
https://proceedings.mlr.press/v162/mishchenko22b.html (ICML 2022)

APDA-Inexact: "Communication Acceleration of Local Gradient Methods via an Accelerated Primal-Dual Algorithm with Inexact Prox"
https://arxiv.org/abs/2207.03957 (NeurIPS 2022)

VR-ProxSkip: "Variance Reduced ProxSkip: Algorithm, Theory and Application to Federated Learning"
https://arxiv.org/abs/2207.04338 (NeurIPS 2022)

RandProx: "RandProx: Primal-Dual Optimization Algorithms with Randomized Proximal Updates"
https://arxiv.org/abs/2207.12891

CompressedScaffnew: "Provably Doubly Accelerated Federated Learning: The First Theoretically Successful Combination of Local Training and Compressed Communication"
https://arxiv.org/abs/2210.13277

GradSkip: "GradSkip: Communication-Accelerated Local Gradient Methods with Better Computational Complexity"
https://arxiv.org/abs/2210.16402

5GCS: "Can 5th Generation Local Training Methods Support Client Sampling? Yes!"
https://arxiv.org/abs/2212.14370

TAMUNA: "TAMUNA: Accelerated federated learning with local training and partial participation"
https://arxiv.org/abs/2302.09832

Pseudo-differential operators on groups

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In this talk we will give an overview of the theory of pseudo-differential operators on Lie groups of different types, and recent advances in this research.

High-Probability Bounds for Stochastic Optimization and Variational Inequalities: the Case of Unbounded Variance

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During recent years the interest of optimization and machine learning communities in high-probability convergence of stochastic optimization methods has been growing. One of the main reasons for this is that high-probability complexity bounds are more accurate and less studied than in-expectation ones. However, SOTA high-probability non-asymptotic convergence results are derived under strong assumptions such as the boundedness of the gradient noise variance or of the objective's gradient itself. In this paper, we propose several algorithms with high-probability convergence results under less restrictive assumptions. In particular, we derive new high-probability convergence results under the assumption that the gradient/operator noise has bounded central α-th moment for $\alpha \in (1, 2]$ in the following setups: (i) smooth non-convex / Polyak-Łojasiewicz / convex / strongly convex / quasi-strongly convex minimization problems, (ii) Lipschitz / star-cocoercive and monotone / quasi-strongly monotone variational inequalities. These results justify the usage of the considered methods for solving problems that do not fit standard functional classes studied in stochastic optimization.

The descent problem for the dual category of ternary rings

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The notion of an effective descent morphism of a category is one of the main notions of Grothendieck's descent theory. The present work is devoted to the descent problem, i.e. the problem of characterizing effective descent morphisms in the category dual to that of ternary rings. We use the term "ternary ring" in the sense of the paper [2] by Machala. A ternary ring is not a ring in the traditional sense, but the category of ternary rings contains that of commutative associative unitary rings as a full reflective subcategory. The category of ternary rings is a variety of universal algebras [1].

In the present work, applying our previous results on effective codescent morphisms in general varieties of universal algebras [3], [4]-[7], we show that every codescent morphism of the variety of ternary rings is effective. This, together with the main result of [4] and the results by Chajda and Halaš on ideals of ternary rings [1], enables us to give the characterization of effective codescent morphisms in the category of ternary rings. Further, the class of morphisms of commutative associative unitary rings which are effective codescent in the variety of ternary rings is compared with that of effective codescent morphisms in the variety of commutative associative unitary rings (recall that the latter morphisms are characterized by the well-known Grothendieck-Joyal-Tierney's criterion). It turns out that the latter class is contained in the former one, but does not coincide with it.

The second author gratefully acknowledges the financial support from Shota Rustaveli National Science Foundation of Georgia (FR-22-4923).

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On the uniform convergence of Fourier series in the double Walsh system by spheres

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The report presents a result related to the question of the uniform convergence of the double Fourier-Walsh series of a.e. finite measurable functions, after changing the values of these functions on a set of small measure. At the same time, the possibility of choosing a corrected function with a spectrum from the reserve of the Fourier-Walsh coefficients of some universal function is considered.

In [1], the following theorem was proved.

Theorem 1. *There exists a (universal) function $U \in L^1[0,1)$, such that for every almost everywhere finite measurable on $[0,1)$ function f and any $\delta > 0$ one can find a (modified) function $g \in L^\infty[0,1)$ with*

$$|\{x \in [0,1) : \ g(x) \neq f(x)\}| \leq \delta,$$

whose Fourier series in the Walsh system converges uniformly on $[0,1)$ and $|c_k(g)| = c_k(U)$, $k \in spec(g)$.

In [2] we have proved the following result.

Theorem 2. *There exists a (universal) function $U \in L^1[0,1)^2$, such that,*

1. *the Fourier-Â-Walsh coefficients of U are positive and strictly decreasing on it's spectrum,*

2. *for every almost everywhere finite measurable on $[0,1)^2$ function f and any $\delta > 0$ one can find a (modified) function $g \in L^\infty[0,1)^2$ with*

$$|\{(x,y) \in [0,1)^2 : \ g(x,y) \neq f(x,y)\}| \leq \delta,$$

whose Fourier-Â-Walsh double series converges uniformly on $[0,1)^2$ by spheres,

3. $|c_{k,s}(g)| = c_{k,s}(U), \ (k,s) \in spec(g).$

In this theorem, the modified function function $g \in L^\infty[0,1)^2$ can be chosen in such a way as to obtain the convergence of the Fourier-Â-Walsh double series both by spheres and by rectangles on $[0,1)^2$.

Note that classes of partial sums (e.g. spherical, rectangular, square) differ sharply from each other when it comes to convergence in $L^p[0,1)^2$, $p \geq 1$, and convergence almost everywhere. Note also, many classical results (for instance, Carleson's, Riesz's and Kolmogorov's theorems cannot be extended from the one-dimensional case to the two-dimensional.

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A free boundary perspective on transmission and inverse scattering problems

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In this talk, I will present some free boundary aspects of transmission and inverse scattering problems. I shall focus on questions related to the regularity theory of solutions and the unknown intefaces, that appear in these problems. The talk will be kept at a narrative and heuristic level.

Towards a Better Theoretical Understanding of Independent Subnetwork Training

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Modern advancements in large-scale machine learning would be impossible without the paradigm of data-parallel distributed computing. Since distributed computing with large-scale models imparts excessive pressure on communication channels, a lot of recent research was directed towards co-designing communication compression strategies and training algorithms with the goal of reducing communication costs. While pure data parallelism allows better data scaling, it suffers from poor model scaling properties. Indeed, compute nodes are severely limited by memory constraints, preventing further increases in model size. For this reason, the latest achievements in training giant neural network models rely on some form of model parallelism as well. In this work, we take a closer theoretical look at Independent Subnetwork Training (IST), which is a recently proposed and highly effective technique for solving the aforementioned problems. We identify fundamental differences between IST and alternative approaches, such as distributed methods with compressed communication, and provide a precise analysis of its optimization performance on a quadratic model.

Convergence of Fourier-Walsh double series in weighted $L^p_\mu[0;1)^2$ spaces

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In this work we will discuss the existence of Walsh double universal series with respect to subseries in weighted $L^p_\mu[0,1)^2$-spaces, the behavior of Fourier coefficients with respect to the Walsh double system, as well as $L^p_\mu[0,1)^2$-convergence of the spherical partial sums of the double Fourier-Walsh series after modification of functions.

The following theorem is hold

Theorem. *There exist a weighted function* $\mu(x;y) : 0 < \mu(x;y) \leq 1, (x,y) \in [0,1)^2$*, and a series by the Walsh double system* $\{\varphi_k(x)\varphi_s(y)\}_{k,s=0}^{\infty}$ *of the form* $\sum\limits_{k,s=0}^{\infty} d_{k,s}\varphi_k(x)\varphi_s(y)$*, with* $\sum\limits_{k,s=0}^{\infty} \left|d_{k,s}\right|^r < \infty$*, for all* $r > 2$ *and non-zero members in* $\{|d_{k,s}|\}_{k,s=0}^{\infty}$*, which are in decreasing order over all rays, with the following property:*

a) for any $\varepsilon > 0$ *there exists a measurable set* $E \subset [0,1)^2$ *with* $|E| > 1-\varepsilon$*, so that for each* $p \in [1,\infty)$ *and for every function* $f(x,y) \in L^p_\mu[0,1)^2$ *one can find a function* $g(x,y) \in L^p_-[0,1)^2 \cap L^1[0,1)^2$ *coinciding with* $f(x,y)$ *on* E*, whose Fourier series* $\sum\limits_{k,s=0}^{\infty} c_{k,s}(g)\varphi_k(x)\varphi_s(y)$ *in the Walsh double system* $\{\varphi_k(x)\varphi_s(y)\}_{k,s=0}^{\infty}$ *converges to the function* $g(x,y)$ *both in the* $L^p_\mu[0,1)^2$*-norm and* $L^1[0,1)^2$*-norm with respect to spheres and*

$$c_{k,s}(g) = d_{k,s},\ for\ all\ (k,s) \in spec(g).$$

b) for each $p \in [1,\infty)$ and for every function $f(x,y) \in L^p_\mu[0,1)^2$ one can find $\delta_{k,s} = 0$ or 1 such that

$$\lim_{R\to\infty} \int_0^1 \int_0^1 \left| \sum_{0\le k^2+s^2\le R^2} \delta_{k,s} d_{k,s} \varphi_k(x)\varphi_s(y) - f(x,y) \right|^p \mu(x;y)dxdy = 0.$$

Note that the statement a) of this Theorem is obtained (is proved) in [1].

This research was supported by the Science Committee of the Republic of Armenia.

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A mathematical model for the optimization problem of bending a variable-thickness orthotropic beam

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In this paper, we consider a non-classical problem of bending an orthotropic beam of variable thickness. An improved theory of orthotropic plates of variable thickness is applied [1]. Based on this theory, a mathematical model for solving the considered physical problem is constructed. This is a system of differential equations with variable coefficients and appropriate boundary conditions. As a boundary condition, a non-classical elastic clamping condition was considered [2]. Optimization was performed depending on the form and type of the applied external load.

The resulting system of differential equations was solved by the collocation method. The calculations were carried out for different values of the parameters. Comparisons were made and based on the analysis, the best option was found according to a certain principle.

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Piecewise linear approximation of numerical solution of two-dimensional boundary value problems

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In the numerical solution of two-dimensional non-linear boundary value problems of mathematical physics, the finite elements method is often used. This method assumes that the domain of the boundary problem is divided into small sub-domains (elements) within which the desired function is assumed to be linear. Thus, the desired solution is approximated by a piecewise linear function. Its graph consists of triangles, the projections of which on the OXY plane form a triangular mesh.

The convergence rate of process of successive approximations to the numerical solution of the problem depends on the geometrical configuration of the corresponding mesh.

We prove the following extremal property of triangular meshes: *The sum of cotangents of interior angles as a function on meshes with fixed set of knots reaches his minimum for Delaunay triangulation.*

Using this extremal property, the theorem is obtained, that for any fixed knots set, for numerical solution of Maxwell equation of electromagnetic field the optimal mesh (in the sense of increasing the rate of convergence) is Delaunay triangulation.

On one nonlinear fourth-order integro-differential parabolic equation

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In [1] reduction of the well-known Maxwell system [2] of differential equations to the form of integro-differential equations was first performed and has a following form

$$\frac{\partial H}{\partial t} = -rot\left[a\left(\int_0^t |rotH|^2 d\tau\right) rotH\right], \tag{1}$$

where, $H = (H_1, H_2, H_3)$ is the vector of the magnetic field. Some qualitative and structural properties of solutions of (1) type systems are established in many works. For more detail information see, for example, [3],

[4] and references therein.

The presented work discusses a natural mathematical generalization of the scalar analog of the integro-differential model (1). In particular, the corresponding fourth-order integro-differential equation is investigated. Several properties of the corresponding initial-boundary value problems are studied.

In the rectangle $Q_T = [0,1] \times [0,T]$, where T is a positive constant, the following problem is considered:

$$\frac{\partial u}{\partial t} + \frac{\partial^2}{\partial x}\left\{\left[1 + \left(\int_0^t \left(\frac{\partial^2 u}{\partial x^2}\right)^2 d\tau\right)\frac{\partial^2 u}{\partial x^2}\right]\right\}, \tag{2}$$

$$u(0,t) = u(1,t) = 0, \tag{3}$$

$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0, \tag{4}$$

$$u(x,0) = u_0(x). \tag{5}$$

In equation (2) and in the initial condition (5), f and u_0 are given functions of their arguments.

The stability and uniqueness of the solution of the initial-boundary value problem (2) - (5) is studied.
where k_m is a coefficient of heat conductivity. This coefficient is a function of Θ as well.

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Monotone Programming: Moments Method for nonparametric classes of distributions

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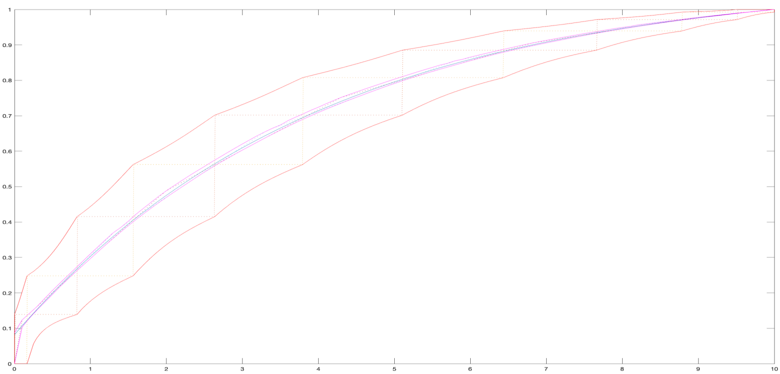
We present a software that reconstructs a cumulative distribution function (CDF) on a finite interval by its first n moments. We do it for any $n \leq 10$, any set of moments $c_1, ..., c_n$ and any finite interval $[a, b]$. If there is no such CDFs, we find the one with maximal possible number of coinciding first moments. If there are many CDFs having such moments we find exact upper and lower limits of values of such CDFs at a fixed point $x \in [a, b]$. Thus, all CDFs with moments $c_1, ..., c_n$ are confined between the limits/bounds. We plot the bounds for each $x \in [a, b]$.

We do the same for some nonparametric classes of CDF e.g. class of all concave CDFs on $[a, b]$. The exact upper and lower bounds are much tighter in this case.

As we are able to do it for any $c_1, ..., c_n$, we can do it for first n empirical moments of a sample and, therefore, we can effectively apply the moments method to estimate the CDF from a nonparametric class. It is not a staircase empirical CDF but a unique CDF from the nonparametric class that have maximal possible first moments coinciding with the empirical ones.

We have developed an original technique we called monotone programming that is a natural continuation of a bisection method in computational mathematics. The latter exists only for monotone functions of one variable. Its beauty is in the fact that it uses only “if” and “$<$“ operations. So do we. We successfully applied monotone programming to Polynomials, Optimal control and, as you see above, Probability and Statistics.

In the example below we take as $c_1, ..., c_{10}$ the first 10 moments of a CDF in blue (exponential). It is a concave one. We calculated the exact upper and lower bounds for values of CDFs for the class of all CDFs on $[0, 10]$ and then for the class of all concave CDFs on $[0, 10]$. As you see in the figure, for the second case the limits are very close to the original CDF in blue.



Gradient-free optimization from noisy data and nonparametric regression

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This talk deals with the problem of estimating the minimizer or the minimal value of a smooth function by exploration of its values under possibly adversarial noise. We consider active (sequential) and passive settings of the problem and various approximations of the gradient descent algorithm, where the gradient is estimated by procedures involving function evaluations at randomized points and a smoothing kernel based on the ideas from nonparametric regression. The objective function is assumed to be either Hölder smooth or Hölder smooth and satisfying additional assumptions such as strong convexity or Polyak–Łojasiewicz condition. In all scenarios, we suggest polynomial time algorithms achieving non-asymptotic minimax optimal or near minimax optimal rates of convergence. The talk is based on a joint work with Arya Akhavan, Evgenii Chzhen, Davit Gogolashvili and Massimiliano Pontil.

On the Fredholm solvability of regular hypoelliptic operators

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Regular hypoelliptic operators form a special subclass of Hyormander's hypoelliptic operators and have many important applications. This class includes elliptic, parabolic, $2b$–parabolic, and quasielliptic operators. The analysis of regular hypoelliptic operators has certain challenges as corresponding characteristic polynomials are not homogeneous like in the elliptic case. While solvability conditions, a priori estimates, and Fredholm properties have been studied for special classes of hypoelliptic operators in various functional spaces, most of the existing results are related to elliptic and quasielliptic operators (see [1, 2, 3, 4]).

We study a priori estimates and Fredholm solvability for a wide class of regular hypoelliptic operators on the special scales of multianisotropic spaces. We establish the normal solvability and a priori estimates for regular hypoelliptic operators with variable coefficients that have certain behavior at infinity. Fredholm criteria are obtained for regular hypoelliptic operators acting in weighted multianisotropic Sobolev spaces in $\mathbb{R}^n$. The considered scales of multianisotropic spaces and conditions on the coefficients are more general than those in previous works (see [5, 6]). Furthermore, we investigate spectral properties, regularity of solutions and index invariance of regular hypoelliptic operators on the special scales of multianisotpic spaces.

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Uniqueness theorems in analysis

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Several theorems in analysis may be treated as uniqueness theorems: the Nyquist–Shannon sampling theorem in signal processing, the F. and M. Riesz theorem in measure theory, Cantor's theorem on trigonometric series, and Carlson's theorem in complex analysis. We explore the interrelations between those theorems.

Lipschitz widths–Remarks

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Dozens of widths are known, see e.g. [1]. Generally they are sequences of numbers that measure the best, *possible under given restrictions,* approximation of the set $\mathcal{K} \subset X$ where $\mathcal{K}$ is (usually) a compact subset of a Banach space X.

In recent decades in numerical analysis we see the growing interest in non-linear algorithms. While it is well known that nonlinear methods of approximation can often perform dramatically better than linear methods, there are still questions on how to measure the optimal performance possible for such methods. Some attempts were made in [2] however they were not taking into account the numerical stability of the methods. In [3] stable manifold width was introduced:

$$\delta_{n,\gamma}(\mathcal{K}) = \inf_{a,M,\|.\|_Y} \sup_{k\in\mathcal{K}} \|f - M(a(k))\|_X$$

where $a : \mathcal{K} \to \mathbb{R}^n$, $M : \mathbb{R}^n \to X$, $\|.\|_Y$ is a norm on $\mathbb{R}^n$ and both a and M are γ-Lipschitz map when on $\mathbb{R}^n$ we use $\|.\|_Y$. This is a stable nonlinear analogue of classical linear width. To get the linear width simply replace here Lipschitz maps into linear operators.

In [4] we introduce Lipschitz widths

$$d_n^{\gamma}(\mathcal{K}) = \inf_{\|.\|_Y} \inf_{\Phi_n} \sup_{k\in\mathcal{K}} \inf_{y\in B_n} \|k - \Phi_n(y)\|_X$$

where $\|.\|_Y$ is a norm on $\mathbb{R}^n$, B_n is a unit ball in $\mathbb{R}^n$ equipped with norm $\|.\|_Y$ and $\Phi_n : B_n \to X$ is a γ-Lipschitz map. This is a nonlinear analogue of classical Kolmogorov widths.

In the talk I will present Lipschitz widths, I will try to present

1. The justification for this width.
2. Basic properties of this new width.
3. Relations between Lipschitz width and classical ones especially the entropy numbers.
4. Present some results and examples motivated by important various numerical procedures specially by deep learning.

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Blocks of representations of periplectic supergroups in positive characteristic

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In my talk I am going to discuss the problem of description of block structure of the categories of rational supermodules over periplectic supergroups. More precisely, if G is an algebraic supergroup, defined over a perfect field of characteristic $p > 2$, then one can introduce the smallest equivalence relation on irreducible G-supermodules, such that two irreducible G-supermodules L and M are equivalent (or linked), provided $Ext^1_G(L, M) \neq 0$ or $Ext^1_G(M, L) \neq 0$. The corresponding equivalence classes are called blocks. If B is a block, then we say that a (not necesary irreducible) G-supermodule V belongs to B, if all composition factors of V belong to B. The category of rational G-supermodules is decomposed into the direct sum of full subcategories, each of which consists of supermodules belonging to the same block.

In my talk I will mentioned the previous results, obtained in collaboration with professor F.Marko (Penn State, US), about blocks over $GL(m|n), OSp(m|2n)$, and recently obtained decription of blocks over $P(n)$.

Chambers and walls in the spaces of real algebraic curves of small degree on a hyperboloid

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Let C be a space of real algebraic curves of a fixed degree on the plane/on a surface, $\Delta \subset C$ be the set of singular curves, $\Delta_1 \subset \Delta$ consist of the curves with a single node or a cusp. The components of $C \setminus \Delta$ (resp. of Δ_1) are *chambers* (resp. *walls*). A path in a chamber (resp. in a wall) is a *rigid isotopy* of the corresponding curves.

In the talk there is an overview of the known results on the rigid isotopy classification of plane curves of degree $m \leq 6$ and curves of small degrees on quadrics. The study of singular real trigonal curves on the Hirzebruch surface Σ_3 with the help of the graphs of these curves is used to complete the rigid isotopy classification of real algebraic curves of bidegree (4,3) begun in [1], [2], the adjacency graph of chambers and walls in the space of these curves is given.

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