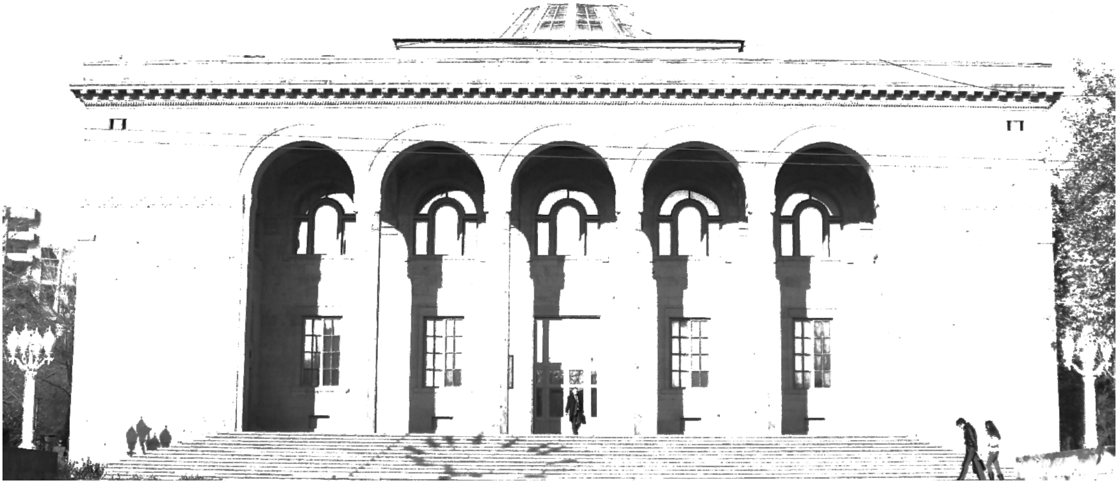

Second International Conference

Mathematics in Armenia

Advances and Perspectives

24 - 31 August 2013, Tsaghkadzor, Armenia

Dedicated to the 70th Anniversary of Foundation of Armenian National Academy of Sciences

ABSTRACTS

Yerevan, 2013

SECOND INTERNATIONAL CONFERENCE

MATHEMATICS IN ARMENIA

ADVANCES AND PERSPECTIVES

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Yerevan, 2013

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Invited Lectures

The Burnside Problem on periodic groups for odd exponents $n > 100$

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In 1902 W. Burnside formulated the following problem:

Is every group generated by a finite number of generators and satisfying the identical relation $x^n = 1$ finite?

Maximal periodic groups $B(r, n)$ with r generators satisfying the identical relation $x^n = 1$ are called *free Burnside groups of exponent n.*

During several decades many mathematicians from different countries studied this problem.

In 1950 W. Magnus formulated a special question on the existence of a maximal finite quotient group of the group $B(r, n)$ for a given pair (r, n). Magnus named this question the "*Restricted Burnside Problem*".

A negative solution of the full (nonrestricted) Burnside problem was given in 1968 by P.S. Novikov and S.I. Adian. It was proved that the groups $B(r, n)$ are infinite for any $r > 1$ and odd $n \geq 4381$.

In 1975 the author published a book where he presented an improved and generalized version of Novikov-Adian theory for odd exponents $n \geq 665$ and established some other applications of the method.

In this talk we introduce a new simplified modification of Novikov-Adian theory that allows to give a shorter proof and stronger results for odd exponents. The main result is the following new theorem.

Theorem 1. *The free Burnside groups $B(m, n)$ are infinite for any odd exponent $n > 100$.*

A detailed survey of investigations on the Burnside Problem and on the Restricted Burnside problem one can find in authors survey paper published in "Russian Math. Surveys", 65:5 (2010), pp. 805 - 855.

On some problems and results in the Weierstrass theory of analytic functions

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We will present a survey of results on possibility of analytic continuation of power series and on efficient restoration of that continuation, on localization of singularities of power series on the boundary of the convergence circle in terms of their coefficients. Also we will outline some analog problems and results on harmonic continuation of Laplace series.

Discrepancy theory and analysis

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Discrepancy theory is a fascinating branch of mathematics which studies different variations of the following question: how well can one approximate continuous distributions by discrete sets of points? And how large are the errors that necessarily arise in such approximations?

Despite the apparently simple formulations, several fundamental problems in the field are still open, especially in higher dimensions. Historically, the methods of functional and harmonic analysis (Fourier analysis, wavelets, Riesz products etc.) have played a pivotal role in the development of the subject. In addition, the subject has profound connections to approximation theory (numerical integration, metric entropy, hyperbolic cross approximations), probability (small deviation problems, empirical processes), combinatorics, and number theory.

In this talk, I will discuss some classical and recent results, methods, and ideas in this field.

Gauss-type operators, intertwining with Hilbert transform, and the Klein-Gordon equation

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This reports on joint work with A. Montes-Rodriguez. We study the (signed) invariant measures for a Gauss-type transformation of an interval, and show how this leads to results concerning the completeness of certain complex exponential systems. We then take things one step further and study invariant distributions from the space $L^1+\mathrm{Hilb}(L^1_0)$, where Hilb is the Hilbert transform and L^1_0 is the codimension 1 subspace of functions with integral 0. This then leads to more refined completeness results.

Asymptotic behaviour of zeros of random polynomials and analytic functions

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For an analytic function G denote by μ_G the measure counting the complex zeros of G according to their multiplicities. Let $\xi_0, \xi_1, \dots$ be non-degenerate independent and identically distributed random variables. Consider a random polynomial

$$G_n(z) = \sum_{k=0}^{n} \xi_k z^k.$$

The first question we are interested in is an asymptotic behaviour of the average number of real zeros of G_n as $n \to \infty$ under different assumptions on the distribution of ξ_0. Afterwards we consider all complex zeros of G_n and study the asymptotic behaviour of random empirical measure μ_{G_n}.

Finally, we consider the generalization of the previous problem to a random analytic function of the following form:

$$G_n(z) = \sum_{k=0}^{\infty} \xi_k f_{k,n} z^k.$$

On Freiman's theorem

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By a celebrated theorem of Freiman, if A is a finite set of reals and $|A+A| \le K|A|$ then there is $d \le d(K)$, positive integers $N_1, ..., N_d$, and an affine mapping $\Phi : \mathbb{R}^d \to \mathbb{R}$ such that $N_1, ..., N_d \le D(K)|A|$ and

$$A \subset \Phi\left(([0, N_1) \times ... \times [0, N_d)) \cap \mathbb{Z}^d\right)$$

It turns out that Freiman's Theorem is very important for numerous applications and very attractive. Currently best results related to Freiman's

Theorem use combination of various ideas from Harmonic Analysis, Combinatorics, Probability Theory and Geometry of Numbers.

Kicked quasi-periodic Schrodinger cocycles and operators

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We will discuss some properties of quasi-periodic 1D Schrodinger cocycles and operators with smooth peaky potentials and show for such potentials coexistence of absolutely continuous and purely continuous parts in the spectrum. (Joint work with K. Bjerklov).

Spectral inequalities for Partial Differential Equations and their applications

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We shall discuss properties of the discrete and continuous spectrum of different classes of self-adjoint differential operators including Schrodinger operators.

Hausdorff dimension and σ-finiteness of some p-harmonic measures

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In this talk for fixed $p, 1 < p < \infty$, we consider a measure $\hat{\mu}$, associated with a positive p-harmonic function $\hat{u}$ defined in an open set $G \subset \mathbb{R}^2$ and vanishing on a portion Γ of ∂G.

During the first part of the talk we will outline results obtained by the presenter and coauthors when G is a bounded simply connected domain in $\mathbb{R}^2$ and $\hat{u}$ is p-harmonic near ∂G with continuous boundary value zero on ∂G. Our results generalize the famous theorem of Makarov concerning the dimension of harmonic measure.

During the second part of this talk we will consider p-harmonic measure in $\mathbb{R}^n, n > 3$. In this case we will outline some examples constructed by the presenter and coauthors. We will also discuss recent attempts by the presenter and coauthors to prove his conjecture concerning the ‘ natural generalization ’ of Jones and Wolff’s theorem for harmonic measure to the p-harmonic setting when $p \geq n$.

Rational polyhedra, unital abelian l-groups, and MV-algebras

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Markov’s celebrated unrecognizability theorem states that there is no algorithm to determine whether or not two polyhedra P and Q in R^n are piecewise linear (PL) homeomorphic. It is understood that the input polyhedra P and Q are explicitly written down as finite unions of simplexes with rational vertices in R^n. In a resource-aware category of rational polyhedra suitable for computations, one may conveniently define arrows as Z-maps, i.e., continuous PL-maps whose linear pieces have integer coefficients. The resulting Z-homeomorphisms then coincide with PL-homeomorphisms that preserve the least common denominator of the coordinates of each rational point. As shown, e.g., in the author’s book “*Advanced Lukasiewicz calculus and MV-algebras*”, Springer 2011, the resulting category is dual to finitely presented MV-algebras, the algebras of Lukasiewicz infinite-valued logic. The equational class of MV-algebras, in turn, is categorically equivalent to unital l-groups (i.e., abelian groups equipped with a translation invariant lattice order, and with an archimedean element), which are a modern mathematization of euclidean magnitudes. Further, via Grothendieck’s K_0 functor, AF C^* -algebras whose Murray-von Neumann order of projections is a lattice correspond to countable unital l-groups. Then invariant measures of rational polyhedra amount to invariant states in their corresponding unital l-groups and AF C^*-algebras. The rich interplay between algebraic, geometric, PL-topological, measure-theoretic and logic-algorithmic notions will be our main concern in this talk.

Wiener's conjecture on cyclic vectors

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N. Wiener characterized cyclic vectors (with respect to translations) in $l^p(Z)$ for $p = 1$ and $p = 2$ in terms of the zero sets of Fourier transform. He conjectured that a similar characterization should be true for $1 < p < 2$. We proved recently in collaboration with Nir Lev that this is not the case. I'll discuss the result and mention some open problems.

Flag algebras

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A substantial part of extremal combinatorics studies relations existing between densities with which given combinatorial structures (fixed size "templates") may appear in unknown (and presumably very large) structures of the same type. Using basic tools and concepts from algebra, analysis and measure theory, we develop a general framework that allows to treat all problems of this sort in an uniform way and reveal mathematical structure that is common for many known arguments in the area. The backbone of this structure is made by certain commutative algebras depending on the problem in question. Once understood, it gives rise to the possibility of computer-aided theorem proving in this area based upon semi-definite programming.

In this talk I will give a general impression of how things work in this framework; a survey of concrete results obtained with the help of the method can be found at `http://people.cs.uchicago.edu/~razborov/files/flag_survey.pdf`

Variations on a problem of Fichera and Stampacchia

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The obstacle problem appeared in the mathematical literature about half a century ago, as a semi-coercive minimization problem considered by Fichera (1963/64) and as a coercive non-symmetric bilinear form on convex sets by Stampacchia (1964). While the first work was motivated by the Signorini problem in elastostatics, the second one started with an application to second order elliptic operators with discontinuous coefficients and potential theory. Having in mind an application to a unilateral thermal membrane problem, we present some recent results for a new variation of this well-known free boundary problem.

On functional-difference operators related to the modular double

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We will discuss the eigenfunction expansion theorem for special functional-difference operators related to Faddeev's quantum modular double and non-compact quantum dilogarithm. This is a joint work with L.D. Faddeev.

On martingales and integrability criteria of trigonometric series

Talalyan A.A. (Institute of Mathematics NAS, Armenia)

We denote by

$$\sum_{\mathbf{n}} c_{\mathbf{n}} e^{2\pi i \mathbf{n}\cdot\mathbf{x}}, \tag{1}$$

$$\mathbf{n} = (n_1, n_2, \ldots, n_m), \quad \mathbf{x} = (x_1, x_2, \ldots, x_m), \quad \mathbf{n}\cdot\mathbf{x} = \sum_j n_j x_j,$$

multiple trigonometric series, possessing one of the properties

1) $|c_{\mathbf{n}}| \leq M$, $\forall \mathbf{n}$, or

2) $|c_{\mathbf{n}}| \le M$ and $\lim_{n_j\to\infty} c_{n_1,\dots,n_j,\dots,n_m} = 0$ for each j and for fixed n_i, $i \ne j$.

Let $\{\chi_\mu(\mathbf{x})\}$, $\mu = (\mu_1, \dots, \mu_m)$,where μ_j runs over all natural numbers, be the multiple Haar system. If $\{Q_\nu\}$ is a sequence of domains such that $Q_1 \subset Q_2 \subset \dots \subset Q_\nu \subset \dots$, $\cup Q_\nu = \mathbb{R}_m$ and Q_ν contains $\mathbf{n}$ and $-\mathbf{n}$ simultaneously, then by S_{Q_ν} we denote the partial sums of series (1) of the form

$$S_{Q_\nu}(x) = \sum_{\mathbf{n}\in Q_\nu} c_{\mathbf{n}} e^{2\pi i \mathbf{n}\cdot\mathbf{x}}.$$

For a given natural k denote by Δ_k^p, $p = (p_1, \dots, p_m)$, $1 \le p_j \le 2^k$, the m-dimensional cube of order k defined by

$$\Delta_k^p = \prod_{j=1}^m \left[\frac{p_j - 1}{2^k}, \frac{p_j}{2^k}\right).$$

We associate with series (1) a multiple series by Haar system

$$\sum_\mu a_\mu(t)\chi_\mu(x) \tag{2}$$

with the coefficients defined by

$$\lim_{\nu\to\infty} \int_{\mathbb{T}} \left| a_\mu(t) - \int_{\mathbb{T}} S_{Q_\nu}(x+t)\chi_\mu(x)dx \right|^2 dt = 0, \quad \mathbb{T} = [0,1)^m. \tag{3}$$

It turns out that the coefficients $a_\mu(t)$ are defined for almost all $t \in \mathbb{T}$ and do not depend on the selection of $\{Q_\nu\}_{\nu=1}^\infty$, provided $|c_{\mathbf{n}}| \le M$.

Theorem 1. *The series* (1) *is a Fourier series if and only if for almost all $t \in \mathbb{T}$ the series* (2) *is a Fourier series of some function $f_t(x)$ in Haar system.*

Applying Theorem 1 and some other theorems we prove

Theorem 2. *Let the series* (1) *satisfies property 2) and the sequence $\{S_{Q_\nu}(x)\}$ has an integrable lower bound, i.e. $S_{Q_\nu}(x) \ge g(x)$ for a.a. x, $g(x) \in L^1(\mathbb{T})$ and suppose some subsequence $\{Q_{\nu_k}\} \subset \{Q_\nu\}$ has the following properties:*

a) If

$$\Phi_{\nu_k}(x,t) = |\Delta_k^p|^{-1} \int_{\Delta_k^p} S_{Q_{\nu_k}}(u+t)du, \quad x \in \Delta_k^p, \tag{4}$$

then for almost all $t \in \mathbb{T}$ the sequence $\Phi_{\nu_k}(x,t)$ converges a.e. to some integrable function $f_t(x)$, $x \in \mathbb{T}$.

b) $\sup_k |\Phi_{\nu_k}(x,t)| < \infty$ for almost all t, and for x outside of a countable union of hyperplanes E_t, which are orthogonal to coordinate axes.
Then (1) *is a Fourier series.*

We prove also some theorems generalizing some classical theorems on integrability of series (1), which $(C,1)$-means have uniformly absolute continuous integrals. We consider also the case, when some conditions are imposed for other means of series (1). In particular, we prove the following result:

Theorem 3. *The series* (1)*, with* $|c_{\mathbf{n}}| \le M$*, is a Fourier series, if and only if some sequence* $\{S_{Q_{\nu_k}}(x)\}$ *have the following properties:*

c) there exist sequences $\varepsilon_k \downarrow 0$*,* $\delta_k \downarrow 0$ *and sets* $E_k \subset \mathbb{T}$*, such that* $|E_k| > |T| - \delta_k$*,* $\forall k$*,*

d) if $\{\Delta_k^p\}$ *is a finite collection of pairwise disjoint cubes of order* k *and* $\sum\limits_p |\Delta_k^p| < \delta_k$*, then*

$$\left| \int_{\cup \Delta_k^p} S_{\nu_k}(x+t)dx \right| < \varepsilon_k, \quad \forall t \in E_k.$$

We use the martingale properties of partial sums of multiple Haar series in the proofs of all these theorems.

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Performance of greedy algorithm in estimating Kolmogorov diameters

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The problem we discuss originates from the reduced basis method introduced for the accurate online evaluation of solutions to a parameter dependent family of elliptic partial differential equations. Abstractly, it can be viewed as determining a "good" n dimensional space $\mathcal{H}_n$ to be used in approximating the elements of a compact set $\mathcal{F}$ in a Hilbert or Banach space $\mathcal{H}$ where solutions live. One, by now popular, computational approach is to find $\mathcal{H}_n$ through a greedy strategy. It is natural to compare the approximation performance of the $\mathcal{H}_n$ generated by this strategy with that of the Kolmogorov widths $d_n(\mathcal{F})$ since the latter gives the smallest error that can be achieved by subspaces of fixed dimension n. The first such comparisons, given in A. Buffa, Y. Maday, A.T. Patera, C. Prud'homme, and G. Turinici, *A Priori convergence of the greedy algorithm for the parameterized reduced basis* M2AN Math. Model. Numer. Anal., **46**(2012), 595–603, show that the approximation error in a Hilbert space, $\sigma_n(\mathcal{F}) := \mathrm{dist}(\mathcal{F}, \mathcal{H}_n)$, obtained by the greedy strategy satisfies $\sigma_n(\mathcal{F}) \leq Cn2^n d_n(\mathcal{F})$. In this talk, various improvements of this result will be given both in Hilbert and in Banach space case. We discuss both individual comparison between $\sigma_n(\mathcal{F})$ and $d_s(\mathcal{F})$ and the estimates for classes when we assume certain decay of $d_n(\mathcal{F})$ and obtain related decay of $\sigma_n(\mathcal{F})$.

This talk reports the joint work with Peter Binev, Albert Cohen, Wolfgang Dahmen, Ronald DeVore and Guergana Petrova.

Real and Complex Analysis

Optimal uniform and tangential approximation by entire and meromorphic functions

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In this talk we discuss the problem of optimal uniform and tangential approximation on the sector $\Delta_\alpha = \{z \in \mathbb{C} : |\arg z| \leq \alpha/2\}$ for $\alpha \in (0, 2\pi)$ by entire and meromorphic functions. The problem of uniform and tangential approximation on the sector by entire functions was studied by H. Kober [1], M.V. Keldysh [2], Mergelyan [3], N. Arakelian [4]-[5] and other authors.

Here we suppose that the approximable function $f \in A'(\Delta_\alpha)$, i. e. $f \in C'(\Delta_\alpha)$ and is holomorphic on the interior of Δ_α. We estimate the growth of the approximating functions on the complex plane depending on the growth of f on Δ_α and the differential properties of f on the boundary of Δ_α.

In the case when the approximating functions are meromorphic, the growth of the approximating functions was represented by their Nevanlinna characteristic. Here also we discuss the set of the poles of the approximating meromorphic functions on the complex plane.

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Approximation of the differentiation operator by bounded operators in the space L^2 on a half-line

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There are many investigations devoted to Stechkin's problem on the best approximation of the differentiation operator of order k on the class of n times differentiable functions by linear bounded operators in the spaces $L_p(I),\ 1 \le p \le \infty$, on the real line $I = (-\infty;\infty)$ and the half-line $I = [0,\infty)$ for $0 \le k < n$; see papers [1, 4] and the references therein. In particular, Yu.N.Subbotin and L.V.Taikov (1967) have solved this problem in the space $L_2(-\infty,+\infty)$ for arbitrary k and n, $0 < k < n.$ In the space $L_2(0,\infty)$, an exact solution is unknown even for $k = 1$ and $n = 2$; the present talk is devoted just to this case.

Denote by $W_2^2 = W_2^2(0,\infty)$ the space of functions $f \in L_2(0,\infty)$ infinitely differentiable on $[0,\infty)$ and such that the derivative f' is locally absolutely continuous on $[0,\infty)$ and the second derivative belongs to $L_2(0,\infty)$. In $W_2^2 = W_2^2(0,\infty)$, we extract the class $Q_2^2 = Q_2^2(0,\infty)$ of functions f such that $\|f''\| \le 1$. Let $\mathscr{B}(N)$ be the set of linear bounded operators in the space $L_2(0,\infty)$ whose norms are bounded by a number $N > 0$: $\|S\|_{L_2\to L_2} \le N$. For an operator $S \in \mathscr{B}(N)$, the value $U(S) = \sup\left\{\|f' - Sf\| : f \in Q_2^2(0,\infty)\right\}$ is the deviation of the operator S form the differentiation operator in the space $L_2(0,\infty)$ on the class Q_2^2. The problem is in studying the value

$$E(N) = \inf\left\{U(S)\colon S \in \mathscr{B}(N)\right\} \tag{1}$$

of the best approximation in the space $L_2(0,\infty)$ on the class Q_2^2 of the differentiation operator by the set $\mathscr{B}(N)$.

Hardy, Littlewood, and Pólya [2, Ch. 7, Sect. 7.8] proved that the sharp inequality $\|f'\|^2 \le 2\,\|f\|\cdot\|f''\|$ holds on the set $W_2^2(0,\infty)$. From this, with the help of arguments originated by S.B.Stechkin, it follows the lower estimate $E(N) \ge 1/(2N)$ of value (1) (see, for example, [1, Sect. 4, formula (4.6)] for details).

Problem (1) was studied by A.L.Rublev and E.E.Berdysheva. Rublev has shown that $E(N) \le 1/(\sqrt[3]{4}N)$. Berdysheva [3] has strengthened this result and proved that

$$E(N) \leq 1/(\sqrt{3}N). \tag{2}$$

During 15 years, this estimate by Berdysheva remained the best known. It was a hypothesis that an equality is valid in (2). However, it turned out that this hypothesis is wrong. The following theorem contains an upper bound for the value of the best approximation (1) that is better than (2).

Theorem 1. *The following estimates are valid for the value of the best approximation* (1)*:*

$$1/(2N) \leq E(N) \leq K/N, \quad K = \sqrt[3]{4}/3. \tag{3}$$

To prove the upper estimate in (3), we use an approximating operator T which is constructed as follows. For a function $f \in L_2(0,\infty)$, consider the differential problem

$$y''' - \alpha y'' - \alpha y' + y = f, \quad y \in L_2(0,\infty), \quad y''(0) = 0,$$

with $\alpha = (2\sqrt[3]{4} + \sqrt[3]{2} - 2)/5$. The operator T is defined by the equality

$$Tf = y' - \alpha y'', \quad f \in L_2(0,\infty).$$

This work is joint with M.A.Filatova [4].

This work was supported by the Russian Foundation for Basic Research (project no. 11-01-00462) and by the Ministry of Education and Science of the Russian Federation (project no. 1.1544.2011).

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On Bergman type projections over Besov and mixed norm spaces

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Let $B = B_n$ be the unit ball in $\mathbb{R}^n$ $(n \geq 2)$, $d\sigma$ be the $(n-1)$-dimensional surface measure on the unit sphere $S = \partial B$, and dV be the volume measure on B. Mixed norm spaces $L(p, q, \alpha)$ are defined to be the sets of those functions $f(x) = f(r\zeta)$ measurable in B, for which the norm

$$\|f\|_{p,q,\alpha} = \Big\|(1-r)^{\alpha} \, \|f(r\cdot)\|_{L^p(S;d\sigma)}\Big\|_{L^q(dr/(1-r))}, \quad 1 \leq p, q \leq \infty, \quad \alpha \in \mathbb{R},$$

is finite.

We generalize some results due to Zhu, Choe, Jevtić, Pavlović and others, on boundedness of Bergman type operators

$$T_\beta(f)(x) = \int_B (1 - |y|^2)^{\beta-1} \, P_\beta(x, y) \, f(y) \, dV(y), \quad x \in B,$$

where P_β is a Poisson-Bergman type kernel function.

With definitions of Besov spaces $\Lambda^{p,q}_\alpha$ of smooth enough functions and its harmonic subspaces $h\Lambda^{p,q}_\alpha$, we find the image of $L(p, q, \alpha)$ with negative α under the mappings T_β.

Theorem 1. *For $1 \leq p, q \leq \infty$, $\alpha \geq 0$, $\beta > 0$, the operator T_β continuously maps the space $L(p, q, -\alpha)$ onto $h\Lambda^{p,q}_\alpha$.*

We also find another Bergman type operator which continuously projects $\Lambda^{p,q}_\alpha$ onto $h\Lambda^{p,q}_\alpha$.

On one singular set for absolute convergence

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First recall the following

Definition 1. *A set $E \subset [0, 2\pi] \times [0, 2\pi]$ is called an absolute convergence set (AC^*-set) for double trigonometric sine series, if the convergence of the series*

$$\sum_{m,n=1}^{\infty} |a_{mn} \sin mx \sin ny|$$

everywhere on E implies

$$\sum_{m,n=1}^{\infty} |a_{mn}| < \infty.$$

A set is called an N^*-set, if it is not an AC^*-set. For general trigonometric series this sets are denoted by AC and N, respectively.

It is proved in [3] that every set of positive measure is an AC-set for multiple trigonometric series. The next result is from [2] (we give it in the simplified form):

Theorem 1. *Let $y = f(x)$, $x \in [0, 2\pi]$ be a continuous monotone function satisfying $f'(x) \neq 0$ a.e. on $[0, 2\pi]$. Then every set consisting of the points of the form $(x, f(x))$, $x \in E \subset [0, 2\pi]$ with $mE > 0$, is an AC^*-set.*

This theorem was extended for general double trigonometric function (under some more restrictive conditions on $f(x)$) in [4].

We will say that the curve L is *singular*, if it is a graph of a continuous, strictly monotone function $y = f(x)$, the derivative of which is zero almost everywhere. The first example of such function was constructed in [1]. [2] contains an example of singular curve, which is an N^*-set.

Here we announce the following result:

Theorem 2. *There exists a singular curve, which is an AC^*-set.*

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Banach-Steinhaus theorem for subspaces

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In the geometric theory of Banach spaces the well known resonance theorem or Banach-Steinhaus theorem states: if the sequence of the norms of operators $A_n : X \to Y$ is unbounded, then there exists a set of second category $V \subset X$, such that for each $x \in V$,

$$sup_{n\in N}\|A_n x|Y\| = \infty.$$

In the classical version of Banach-Steinhaus theorem the spaces and operators are linear. We consider the following problem:

Problem. A. *Suppose that we are given two Banach spaces X, Y and a sequence of linear operators $A_n : X \to Y$ satisfying*

$$sup_{n\in N}\|A_n : X \to Y\| = \infty.$$

Does there exists a subspace $Z \subset X$ of the dimension greater than 1 (infinite-dimensional subspace $Z \subset X$) such that for all $x \in Z$, $x \neq 0$,

$$sup_{n\in N}\|A_n x|Y\| = \infty?$$

Theorem 1. *Let X, Y be Banach spaces and the sequence of operators $A_n : X \to Y$ satisfy*

$$\|A_n|X \to Y\| = \alpha_n < \infty,$$

$$\sup_n \alpha_n = \infty,$$

and for some subspace $Z \subset X$, $\forall\ z \in Z$

$$\sup_n \|A_n z|Y\| < \infty,$$

$$\sup_{z\in Z;\|z|X\|\leq 1} \|A_n z|Y\| = \alpha_n.$$

Then there exists a subspace of infinite dimension $X_0 \subset X$ such that $\forall$ $x \in X_0$, $x \neq 0$ the equality

$$\sup_n \|A_n x|Y\| = \infty$$

is valid.

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Signs and permutations: Two problems of the function theory

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We apply the technique of the transference lemma

a) to get a refined version of the Garsia inequality for orthogonal systems (the case $p \geq 2$);
b) to show that the (σ,θ)-condition on the Fourier series of a continuous 2π-periodic function f implies the uniform convergence of a rearrangement of the series to f.

The talk is based on [1].

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Averaging of spectral measures associated with the Weyl-Titchmarsh m-function

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We consider averages of spectral measures of the form

$$\kappa(A) = \int_a^b \rho_\theta(A)d\nu(\theta),$$

where $\{\rho_\theta\}$ is a family of spectral measures associated with the Weyl-Titchmarsh m-function for the Schrödinger equation on the half-line, and ν is an arbitrary Herglotz measure. We show that the measure κ corresponds to a composition of Herglotz functions, and we examine the properties of κ by considering the boundary values of the functions undergoing composition. We give precise conditions for absolute continuity and the discrete part of κ.

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Polynomial approximations on the closed subsets of the unit circle and applications

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Let F be an arbitrary closed subset on the unit circle T in the complex plane, and let f be a continuous complex valued function on F. We consider the problem of uniform approximation of f on F by (analytic) polynomials P_n that are uniformly bounded on T by a positive number M ($M = M_f$); the problem is asking to find the necessary and sufficient conditions on f providing the possibility of such approximation of f. In a particular case when F is a closed arc of T, the problem has been solved by L. Zalcman in 1982. We presents the complete solution of the problem in the general case. Some further questions will be discussed as well. In particular, as an application of the main result we derive a new proof for the classical interpolation theorem of W. Rudin and L. Carleson. The new approach also allows to give an alternative simple proof for E. Bishop's peak-interpolation theorem.

Nikol'skii inequality for algebraic polynomials on a sphere and an interval

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Let $\mathbb{R}^m,\ m \geq 2$, be the Euclidean space with the inner product $xy = \sum_{k=1}^m x_k y_k,\ x = (x_1, x_2 \ldots, x_m),\ y = (y_1, y_2, \ldots, y_m)$, and the norm $|x| = \sqrt{xx}$. Let $\mathbb{S}^{m-1} = \{x \in \mathbb{R}^m : |x| = 1\}$ be the unit sphere of the space $\mathbb{R}^m$ centered at the origin. Denote by $\mathcal{P}_{n,m}$ the set of algebraic polynomials $P_n(x) = \sum c_\alpha x^\alpha,\ \alpha_1 + \ldots + \alpha_m \leq n,\ \alpha = (\alpha_1, \ldots, \alpha_m) \in \mathbb{Z}_+^m,\ x^\alpha = x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_m^{\alpha_m}$ of degree (at most) n in m (real) variables with real coefficients c_α.

The main goal of this talk is the best constant $C(n,m)_q$ in the inequality

$$\|P_n\|_{C(\mathbb{S}^{m-1})} \leq C(n,m)_q \, \|P_n\|_{L_q(\mathbb{S}^{m-1})}, \quad P_n \in \mathcal{P}_{n,m}, \quad 1 \leq q < \infty, \quad (1)$$

between the uniform norm and the integral q-norm of a polynomial of given degree on the unit sphere. We will say that a polynomial P_n^* is the unique extremal polynomial in inequality (1) if this polynomial is extremal in (1) (i.e., (1) turns into an equality for this polynomial) and the restriction of any other extremal polynomial P_n^{**} to the sphere is representable in the form $P_n^{**}(x) = cP_n^*(\mathcal{A}x),\ x \in \mathbb{S}^{m-1}$, where $\mathcal{A}$ is an orthogonal transformation of the space $\mathbb{R}^m$ and $c \in \mathbb{R},\ c \neq 0$.

For a weight v and a parameter $q,\ 1 \leq q < \infty$, consider the space $L_q^v(-1,1)$ of functions f measurable on the interval $(-1,1)$ and such that the function $|f|^q$ is summable on $(-1,1)$ with weight v. This is a Banach space with respect to the norm $\|f\|_{L_q^v(-1,1)} = \left(\int_{-1}^1 |f(t)|^q \, v(t) \, dt \right)^{1/q}$, $f \in L_q^v(-1,1)$.

Denote by $M(n,\phi)_q$ and $\widetilde{M}(n,\phi)_q$ the least constants in the inequalities

$$\|p\|_{C[-1,1]} \leq M(n,\phi)_q \, \|p\|_{L_q^\phi(-1,1)}, \qquad p \in \mathscr{P}_n = \mathscr{P}_{n,1}, \qquad (2)$$

$$|p(1)| \leq \widetilde{M}(n,\phi)_q \, \|p\|_{L_q^\phi(-1,1)}, \qquad p \in \mathscr{P}_n, \qquad (3)$$

respectively, where $\phi(t) = (1-t^2)^\alpha,\ \alpha = \frac{m-3}{2}$, is the ultraspherical weight.

For $n \geq 1$, let $\mathscr{P}_n^1$ be the set of algebraic polynomials in one variable of degree n with leading coefficient equal to 1. Denote by ϱ_n the polynomial from $\mathscr{P}_n^1$ that deviates least from zero in the space $L_q^\psi(-1,1)$ with weight $\psi(t) = (1-t)\phi(t)$; i.e.,

$$\min\{\|p_n\|_{L_q^\psi(-1,1)} : p_n \in \mathscr{P}_n^1\} = \|\varrho_n\|_{L_q^\psi(-1,1)}. \qquad (4)$$

Theorem 1. *For $n \geq 1$, $m \geq 3$, and $1 \leq q < \infty$, the following assertions are valid.*

1. *The best constants in inequalities* (1), (2), *and* (3) *are connected by the following relations:*

$$\widetilde{M}(n,\phi)_q = M(n,\phi)_q = |\mathbb{S}^{m-2}|^{1/q}\, C(n,m)_q, \qquad (5)$$

where $|\mathbb{S}^{m-2}|$ is the Lebesgue measure of the unit sphere from the space $\mathbb{R}^{m-1}$.

2. *The polynomial ϱ_n that deviates least from zero on $(-1,1)$ with respect to the norm of the space $L_q^{\psi}(-1,1)$, as zonal polynomial in one variable $t = x_m$, $x = (x_1, x_2, \ldots, x_m) \in \mathbb{S}^{m-1}$, is the unique extremal polynomial in inequality* (1).

3. *The polynomial ϱ_n is the unique extremal polynomial both in inequality* (3) *and in inequality* (2).

This work is joint with V.V.Arestov [1]. The results for $q = 1$ were obtained earlier in [2].

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L^1 and uniform convergence of series by the generalized Walsh system

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Let a be a fixed integer, $a \geq 2$ and $\omega_a = e^{\frac{2\pi i}{a}}$.

First we give the definitions of generalized Rademacher and Walsh systems.

Definition 1. *The Rademacher system of order a is defined by*

$$\varphi_0(x) = \omega_a^k \quad if \quad x \in \left[\frac{k}{a}, \frac{k+1}{a}\right), \quad k = 0, 1, ..., a-1,$$

and for $n \geq 0$

$$\varphi_n(x+1) = \varphi_n(x) = \varphi_0(a^n x).$$

Definition 2. *The generalized Walsh system of order a is defined by*

$$\psi_0(x) = 1,$$

and if $n = \alpha_1 a^{n_1} + ... + \alpha_s a^{n_s}$ *with* $n_1 > ... > n_s$*, then*

$$\psi_n(x) = \varphi_{n_1}^{\alpha_1}(x) \cdot ... \cdot \varphi_{n_s}^{\alpha_s}(x).$$

The basic properties of the generalized Walsh system of order a are obtained by H.E.Chrestenson, J. Fine, C. Vatari, N. Vilenkin and others. Let's denote the generalized Walsh system of order a by Ψ_a. Note that Ψ_2 is the classical Walsh system.

In this talk we discuss the problem of uniform and L^1 convergence of Fourier series of functions with respect to the generalized Walsh system, after modifying the function on a small set.

Note that Luzin's idea of modification of a function improving its properties was substantially developed later on.

In 1939, Men'shov proved the fundamental theorem (the C-strong property of function). Further interesting results in this direction were obtained by many famous mathematicians. In 1991 M. Grigorian proved the L^1-strong property for the trigonometric and classical Walsh systems.

In [1] we prove the following:

Theorem 1. *For any* $0 < \varepsilon < 1$ *and any* $f \in L^1[0,1)$ *one can find a function* $g \in L^1[0,1)$, $mes\{x \in [0,1) \ ; \ g \neq f\} < \varepsilon$*, such that its Fourier series by the system* Ψ_a *converges in* L^1 *to* $g(x)$ *and nonzero Fourier coefficients are monotonically decreasing by absolute values.*

Moreover, the following is true:

Theorem 2. *For any measurable, almost everywhere finite function* $f(x)$ *on* $[0,1)$ *and any* $0 < \epsilon < 1$*, there exists a function* $g(x) \in L^\infty[0,1)$ *with* $|\{x \in [0,1) : \quad g(x) \neq f(x)\}| < \varepsilon$ *such that its Fourier series by* Ψ_a*,* $a \geq 2$*, converges to* $g(x)$ *uniformly on* $[0,1)$*, and the sequence of coefficients* $\{|c_k(g)| : \quad k \in spec(g)\}$ *is decreasing.*

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On the negative order Cesaro means of the Fourier series in the trigonometric and Walsh systems

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Let $S_n(x,f), n = 0,1,...$ be the partial sums of the Fourier series in the trigonometric system of an integrable function $f(x)$. For a real number $\alpha > -1$ the sums

$$\sigma_n^\alpha(x,f) = \frac{1}{A_n^\alpha} \cdot \sum_{k=0}^{n} A_{n-k}^{\alpha-1} S_k(x,f),\ n = 0,1,...$$

are called the α-th order Cesaro means or (C,α) means of the Fourier series of $f(x)$, where

$$A_0^\alpha = 1,\quad A_k^\alpha = \frac{(\alpha+1)(\alpha+2)...(\alpha+k)}{k!},\ k = 1,2,....$$

In particular, the $(C,0)$ means coincide with partial sums, i.e. $\sigma_n^0(x,f) = S_n(x,f), n \in N$. The Cesaro means of the Fourier-Walsh series are defined similarly.

According to the classical theorem of Fejer-Lebesgue, the positive order Cesaro means of the Fourier series in the trigonometric system of any integrable function converge almost everywhere. J. Fine proved an analogous theorem for the Walsh system. $(C,0)$ means of the Fourier series of an integrable function can diverge almost everywhere. On the other hand, it is known that $(C,0)$ means of Fourier series of any integrable function in the trigonometric system or Walsh system possess an almost everywhere convergent subsequence. In connection with this, for the trigonometric system D.E.Menshov [1] stated a problem: will the statement remain true if the ordinary convergence is replaced by the negative order Cesaro summation methods? In other words, do there exist an almost everywhere convergent subsequence of the negative order Cesaro means of the Fourier series of any integrable function? Besides, in [1] the following theorem is proved.

Theorem. ***M (D.E.Menshov)*** *There exist an integrable function such that any subsequence of the α-th order Cesaro means ($\alpha \in (-1,0)$) sequence of the Fourier series of that function is divergent on a set of positive measure.*

In [2] the author proved an analogous theorem for the Walsh system.

In this report we will present a generalization of Theorem M, and also we will investigate some properties of negative order Cesaro means of the Fourier series in the trigonometric system and in the Walsh systems with monotone coefficients, in particular, we will consider the convergence of negative order Cesaro means of these series in $L_p, p > 1$ metrics.

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On self-commutators of some operators

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It is known that for any Hilbert space operator A the self-commutator $AA^* - A^*A$ satisfies the following inequality

$$\|AA^* - A^*A\| \leq \|A\|^2 .$$

This inequality can be sharpened.

Proposition. *Let $N(A)$ be the set of all normal operators, commuting with A. Then*

$$\|A^*A - AA^*\| \leqslant \inf_{M \in N(A)} \|A + M\|^2 .$$

We calculate the both sides of this expression for

a) the Volterra integration operator in $L^2(0\,;1)$ defined by the formula

$$(Vf)(x) = \int_0^x f(t)\,dt,$$

b) the Hardy operator in $L^2(0\,;1)$ defined by the formula

$$(Hf)(x) = \frac{1}{x}\int_0^x f(t)\,dt,$$

c) the operator of the multiplication by the independent variable in the Bergman space of analytic in the unit disk functions, square integrable with respect to the ordinary Lebesgue plane measure.

General Franklin periodic systems as bases in B^1

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The space B^1 was introduced in [1]. Let T be the unit circle.

Definition 1. *A real-valued function b, defined on T, is a special atom, if either $b(t) \equiv \dfrac{1}{2\pi}$ or $b(t) = \dfrac{1}{|I|}\chi_L(t) - \dfrac{1}{|I|}\chi_R(t)$, where I is an interval on T, L is the left half of I and R is the right half.*

Definition 2. *The function space B^1 is defined by*

$$B^1 = \{f : T \to R; \quad f(t) = \sum_{n=1}^{\infty} c_n b_n(t), \quad \sum_{n=1}^{\infty} |c_n| < \infty\},$$

where b_n are special atoms. Denote $\|f\|_{B^1} = \inf \sum_{n=1}^{\infty} |c_n|$, where the infimum is taken over all representation of f.

It is known that $B^1 \subset H^1 \subset L^1$ and $B^1 \neq H^1 \neq L^1$.

The definitions of general Franklin periodic systems, strong regularity and strong regularity for pairs of partitions $\mathcal{T}$ of $[0;1]$ were given in [2], [3].

Theorem 1. *Let $\mathcal{T}$ be a partition of $[0;1]$ with the corresponding Franklin system $\{f_n\}_{n=0}^{\infty}$. Then $\{f_n\}_{n=0}^{\infty}$ is a basis in B^1 if and only if $\mathcal{T}$ satisfies the strong regularity condition for pairs.*

Theorem 2. *Let $\mathcal{T}$ be a partition of $[0;1]$ with the corresponding Franklin system $\{f_n\}_{n=0}^{\infty}$. Then $\{f_n\}_{n=0}^{\infty}$ is an unconditional basis in B^1 if and only if $\mathcal{T}$ satisfies the strong regularity condition.*

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On general Franklin systems on R^1

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By a general Franklin system on R^1 we mean an orthonormal system, which n-th function is a spline of order 2 having $n+1$ knots. Obviously, these functions vanish outside of the minimal interval containing the corresponding knots. This definition is analogous to the one for general Franklin system. It turns out that the Dirichlet kernel for this system have uniformly bounded L_1 norms and

$$\lim_{n\to\infty}\int_{|x-t|>\delta}|K_n(x,t)|dt=0,\ \forall\,\delta>0.$$

From these properties one can deduce that for any compactly supported function $f\in C(R)$ the corresponding partial sums of Fourier series $S_n(f,x)$ uniformly converge to $f(x)$ on R, and also basisness of general Franklin system on R^1 in $L_p(R)$, for any $p\in(1,\infty)$ and any admissible sequence.

Let $\varphi(x)$ be even function, increasing on $[0,\infty)$. Define

$$C_\varphi(R)=\{f\in C(R):|f(x)|\le c\varphi(x),\quad \text{for some } c>0 \text{ and } \forall x\in R\}.$$

Now consider a partition $\mathcal{T}$ constructed in the following way. Take $t_0=0$, $t_1=-1$, $t_2=1$. In the second step add points $t_3=-2$, $t_4=-\frac{1}{2}$, $t_5=\frac{1}{2}$, $t_6=2$. And in the n-th step let $t_{2^n-1}=-n$, and then successively add from the left to the right the midpoints of intervals obtained by the points defined before the n-th step, and define $t_{2^{n+1}-2}=n$. Continuing in that way we will get a sequence $\mathcal{T}$, which will be dense in R. Let $\{f_n(x)\}_{n=2}^\infty$ be the corresponding Franklin system. Then the following theorems are true:

Theorem 1. *Suppose*

$$\lim_{n\to\infty}\frac{\ln\varphi(n)}{2^n}=0.$$

Then for any function $f\in C_\varphi(R)$ the partial sums $S_n(f,x)$ of Fourier-Franklin series of the function f local uniformly converge to $f(x)$.

Theorem 2. *If $\overline{\lim}_{n\to\infty}\ln\varphi(n)/(2^n)>0$, then there exists a function $f\in C_\varphi(R)$ such that $S_n(f,x)$ don't converge to $f(x)$ at some points.*

By the methods developed in [1] it can be also proved the following theorem:

Theorem 3. *For any admissible sequence on R the corresponding Franklin system on R^1 is an unconditional basis in $L_p(R)$, $1<p<\infty$.*

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On fractional derivatives of trigonometric polynomials

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Let T_n be the set of trigonometric polynomials

$$f_n(t) = a_0 + \sum_{k=1}^{n} (a_k \cos kt + b_k \sin kt), \quad a_k, b_k \in \mathbb{R},$$

and $D^\alpha_\beta f_n$ be the Weyl fractional derivative of order $\alpha \in \mathbb{R}$ with shift β of a polynomial f_n,

$$D^\alpha_\beta f_n(t) = \sum_{k=1}^{n} k^\alpha \big(a_k \cos(kt + \alpha\pi/2 + \beta) + b_k \sin(kt + \alpha\pi/2 + \beta)\big).$$

We will discuss sharp estimates of the L_p-norm of $D^\alpha_\beta f_n$ via the uniform norm of f_n and via the L_p-norm of f_n for $0 \le p < 1$. In particular, we show that the sharp inequality $\|D^\alpha_\beta f_n\|_p \le n^\alpha \|\cos t\|_p \|f_n\|_\infty$ holds for $\alpha \ge 1$, $\beta \in \mathbb{R}$ and $0 \le p < 1$. For $p = \infty$, $\alpha \ge 1$ and $\beta = 0$, this fact was proved by Lizorkin (1965). For $1 \le p < \infty$, positive integer α and $\beta \in \mathbb{R}$, the inequality was proved by Taikov (1965); however, in this case, the inequality follows from results of an earlier paper by Calderon and Klein (1951). For $1 \le p < \infty$, $\alpha \ge 1$, $\beta \in \mathbb{R}$, it was proved by Kozko (1998).

The results were obtained by the author jointly with V.V. Arestov.

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Boundedness of Stein's spherical maximal function in variable Lebesgue space and application to the wave equation

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This is a joint work with Alberto Fiorenza and Tengiz Kopaliani.

Define the Spherical Maximal operator $\mathcal{M}$ by

$$\mathcal{M}f(x) := \sup_{t>0} |u_t * f(x)|,$$

where u_t denotes the normalized surface measure on the sphere of center 0 and radius t in $\mathbb{R}^n$.

We say that a function $p : \mathbb{R}^n \to (0, \infty)$ is locally log-Hölder continuous on $\mathbb{R}^n$ if there exists $c_1 > 0$ such that

$$|p(x) - p(y)| \leq c_1 \frac{1}{\log(e + 1/|x-y|)}$$

for all $x, y \in \mathbb{R}^n$. We say that $p(\cdot)$ satisfies the log-Hölder decay condition, if there exist $p_\infty \in (0, \infty)$ and a constant $c_2 > 0$ such that

$$|p(x) - p_\infty| \leq c_2 \frac{1}{\log(e + |x|)}$$

for all $x \in \mathbb{R}^n$. We say that $p(\cdot)$ is globally log-Hölder continuous in $\mathbb{R}^n$ ($p(\cdot) \in \mathcal{P}_{\log}$), if it is locally log-Hölder continuous and satisfies the log-Hölder decay condition.

Denote $p^- = \operatorname{ess\,inf}_{x\in\mathbb{R}^n} p(x)$ and $p^+ = \operatorname{ess\,sup}_{x\in\mathbb{R}^n} p(x)$.

Theorem 1. *If $p(\cdot) \in \mathcal{P}_{\log}$ and $\frac{n}{n-1} < p^- \leq p^+ \leq p^-(n-1)$, then*

$$\|\mathcal{M}f\|_{p(\cdot)} \lesssim \|f\|_{p(\cdot)}.$$

The result is then interpreted as the preservation of the integrability properties of the initial velocity of propagation to the solution of the initial-value problem for the wave equation.

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Convergence of logarithmic means of Multiple Fourier series

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The maximal Orlicz spaces such that the mixed logarithmic means of multiple Fourier series for the functions from these spaces converge in measure and in norm are found.

Toeplitz operators on weighted Besov spaces of holomorphic functions on the polydisk

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Let S be the space of functions of regular variation, $\omega \in S$. We characterize those symbols h for which the corresponding Toeplitz operators are bounded on weighted Besov spaces $B_p(\omega)$ of holomorphic functions on the polydisk for some $1 \leq p < \infty$. As an application, a division theorem on "good inner" functions is obtained in $B_p(\omega)$.

This is a joint work with W. Lusky.

Uniform continuity and weak almost periodicity on locally compact quantum groups

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For a locally compact group G, a bounded continuous function f is left uniformly continuous if the map $t \mapsto L_t f$, for $t \in G$, is continuous at the identity of the group. Here $L_t f$ is the left translation of f by $t \in G$. The set of all left uniformly continuous functions on G which is a C^*-subalgebra of $C_b(G)$ is denoted by $LUC(G)$. By Cohen factorization we have $LUC(G) = L^\infty(G).L^1(G)$.

For a locally compact quantum group $\mathbb{G}$ in 2008 Runde defined the space $LUC(\mathbb{G})$ of left uniformly continuous and the space $\mathcal{WAP}(\mathbb{G})$ of weakly almost periodic elements in $L^\infty(\mathbb{G})$ and showed that $\mathcal{WAP}(\mathbb{G})$ contains $C_0(\mathbb{G})$ and, whenever $\mathbb{G}$ is co-amenable, is contained in $LUC(\mathbb{G})$. We introduce a different approach to uniform continuity and (weak) almost periodicity, using completely bounded multipliers of $L^1(\mathbb{G})$. We show that our definitions are equivalent to Runde's definitions, provided that $\mathbb{G}$ is co-amenable. We prove that in this new version every weakly almost periodic element in $L^\infty(\mathbb{G})$ is uniformly continuous, for every locally compact quantum group $\mathbb{G}$.

Phase unwrapping problem

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Phase unwrapping is a classical problem which arises from many branches of applied physics and engineering, such as optical interferometry, x-ray phase contrast imaging, MRI, and synthetic aperture radar, etc. In these applications, true phase values from an imaged object are "wrapped" in one principal range such as $(-\pi, \pi]$. This results in not only discontinuities in the measured phase values, but also ambiguities of phase values. Phase unwrapping problem is to restore true phase values from the measured phase data which are wrapped and corrupted by noise. Phase unwrapping is essentially ill-posed due to the ambiguity resulted from the wrapping operator. In our previous work, we developed algorithms for 2-dimensional phase unwrapping problem. One is based on the integer optimization approach and one is based on the Neumann problem of the Poissons equation.

In this talk, we will discuss the sampling issues with the phase unwrapping problem. We have established a sampling formula when the phase gradient is band-limited and its gradient is less than -times of the Nyquist frequency. When this sampling condition on the gradient fails, or when the phase is not band-limited, we have estimated reconstruction errors for band-limited, or exponentially or polynomially decayed phases, respectively, in terms of the sampling interval.

This is a joint work with Wenchang Sun from Nankai University.

On the divergence sets of Walsh-Fourier series of continuous functions

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In 1965 Kahane and Katznelson [3] proved that for any set $E \subset \mathbb{T}$ of Lebesgue measure zero there exists a continuous complex valued function, which Fourier series diverges on E. Using the basic lemma of [3], in 1970 Buzdalin [2] constructed a real valued function with the same property. The investigations of the analogous problem for the Walsh-Fourier series were started in the papers [6], [7], [8], [1]. In [6] Simon constructed a continuous function which Walsh-Fourier series diverges on an uncountable set of measure zero. Harris and Wade [7] constructed a perfect set of zero measure which is a set of divergence for a Walsh-Fourier series of continuous function. Bugadze [1] proved that for a given set E of measure zero there exists a bounded function which Walsh-Fourier series diverges at any point of E. Similar theorems for general sequences of operators were considered in [4], [5]. We prove

Theorem. *For any set $E \subset [0,1]$ of zero measure there exists a continuous function on $[0,1]$, which Walsh-Fourier series diverges at any point $x \in E$.*

The proof of the theorem is constructive and applicable also for the trigonometric and Vilenkin systems, while in the method of Kahane and Katznelson analytic functions were used.

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On β-uniform algebras $H^\infty_\beta(\Delta)$

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Let $C_b(\Delta)$ be the Banach algebra of all bounded, continuous, complex valued functions with sup-norm on the open unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$, and $C_0(\Delta)$ be the ideal in $C_b(\Delta)$ consisting of functions whose limit is zero at the boundary $T = \{z \in \mathbb{C} : |z| = 1\}$.

If we introduce the topology in the algebra $C_b(\Delta)$ with the help of the family of semi-norms $\{P_g\}_{g\in C_0(\Delta)}$, where $P_g(f) = \sup\limits_{\Delta} |fg|$, then the resulting topological algebra, which we will denote by $C_\beta(\Delta)$, is complete in this topology. Recall that a closed subalgebra $\mathcal{A}(\Delta)$ of the algebra $C_\beta(\Delta)$ is called β-uniform, if it contains constants and separates the points of Δ. Since the topology of sup-norm is stronger that the β-uniform topology, it follows that a β-uniform algebra is a closed subalgebra of the Banach algebra $C_b(\Delta)$ with sup-norm.

In what follows $\mathcal{A}_\beta(\Delta)$ is the algebra $\mathcal{A}(\Delta)$ equipped with β-uniform topology, and denote by $A_\infty(\Delta)$ the algebra $\mathcal{A}(\Delta)$ equipped with sup-norm topology. Denote also by $M_{A_\beta(\Delta)}$ the set of all β-continuous linear multiplicative functionals on β-uniform algebra $A_\beta(\Delta)$ and by $M_{A_\infty(\Delta)}$ the space of maximal ideals of uniform algebra $\mathcal{A}_\infty(\Delta)$. Clearly, $M_{A_\beta(\Delta)} \subset M_{A_\infty(\Delta)}$. If $\partial A_\infty(\Delta)$ is the Shilov boundary for algebra $A_\infty(\Delta)$, then we call a Shilov β-boundary for algebra $A_\beta(\Delta)$ the set $\partial A_\infty(\Delta) \cap \Delta$ and denote it by $\partial A_\beta(\Delta)$ (see [1, 2, 3]).

Let $A_\beta(\Delta) = H^\infty_\beta(\Delta)$, then the following statements are true:

1. The algebra $H^\infty_\beta(\Delta)$ is a β-uniform subalgebra in $C_\beta(\Delta)$;

2. $M_{H^\infty_\beta(\Delta)} = \Delta$;

3. The Shilov β-boundary for the β-uniform algebra $H^\infty_\beta(\Delta)$ is empty: $\partial H^\infty_\beta(\Delta) = \emptyset$;

4. There exists a linear multiplicative functional $\varphi : H^\infty_\beta(\Delta) \to \mathbb{C}$, which is not continuous in the β-topology.

In connection with the corona problem we will say that the β-uniform algebra $A_\beta(\Delta)$ has no corona, if $M_{A_\beta(\Delta)}$ is dense in a $*$-weak topology in $M_{A_\infty(\Delta)}$. It's known (see [4]) that a unit disk Δ can be homeomorphly imbedded in $M_{H^\infty(\Delta)}$. The famous Carleson theorem about corona (see [5]) says that the disk Δ is dense in $M_{H^\infty(\Delta)}$. Using statements 2 and 3, we obtain the following assertion:

5. The β-uniform algebra $H_\beta^\infty(\Delta)$ has no corona.

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Weighted integral representations in tube domain over real unit ball

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Let $B = \{(y_1, y_2, \dots, y_n) \in R^n : |y|^2 \equiv y_1^2 + y_2^2 + \dots + y_n^2 < 1\}$ be the real unit ball in R^n and $\gamma(y) > 0, y \in B$, be a continuous function of the class $L^1(B; dy)$. For $1 \le p \le 2, 1/p \le s < +\infty$ we consider the space of functions f holomorphic in the tube domain $T_B = \{z = x + iy \in C^n : x \in R^n, y \in B\}$ and satisfying the condition

$$\int_B \left(\int_{R^n} |f(x+iy)|^p dx \right)^s \cdot \gamma(y) dy < +\infty.$$

For these classes the following integral representation is obtained ($z \in T_B$):

$$f(z) = \frac{1}{(2\pi)^n} \int_{T_B} f(w) \Phi(z,w) \gamma(v) du dv \quad (w = u + iv),$$

where the kernel $\Phi(z,w)$ is holomorphic in z, antiholomorphic in w and is written in the following explicit form:

$$\Phi(z,w) = \int_{R^n} \frac{e^{i<z-\overline{w},t>}}{\gamma^*(2t)} dt, \quad z, w \in T_B,$$

where

$$\gamma^*(t) \equiv \int_B e^{-<t,y>} \cdot \gamma(y) dy, \quad t \in R^n.$$

Moreover, if $0 < \delta < 2$, then

$$|\Phi(z,w)| \leq \frac{const(\delta;\gamma;n)}{\delta^n}$$

uniformly in $z = x + iy, w = u + iv \in T_B$ with $|y| + |v| \leq 2 - \delta$.

On directionally differentiable selections of set-valued mappings

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In the paper, with the help of directionally differentiable selections, derivatives of some set-valued mappings with convex and nonconvex images are constructed. A basic result on the existence of continuous selections is the Michael's theorem [3] concerning the case of lower semi-continuous mappings with closed convex images. In the paper we prove a theorem that is more precise than the Michael's theorem for some classes of set-valued mappings.

By $B_\varepsilon(x)$ we denote the closed ball of radius ε centered at x. If $a : R^n \to R^{2^m}$ is a set-valued mapping, then

$$graf(a) = \{(x,y) : y \in a(x)\}, \quad dom(a) = \{x : a(x) \neq \emptyset\}.$$

A selection for a set-valued mapping $a : R^n \to R^{2^m}$ is defined as a single-valued map $y(x)$ such that $y(x) \in a(x)$ for all $x \in dom(a)$.

Definition 1. *[4] Let M be a subset of R^n. Then the set*

$$M^0 = \{x \in M : \forall y \in M \ \lambda x + (1-\lambda)y \in M, \ \forall \lambda \in [0,1]\}$$

is called the starlike kernel of M. If $M^0 \neq \emptyset$ then M is called a starlike set. We denote by a^0 the set-valued mapping, the graph of which represents the set $(graf(a))^0$.

Definition 2. *[5]. The contingent cone $K_M(x)$ to the set M at $x \in M$ is the upper topological limit of the form*

$$K_M(x) = \lim_{\lambda \downarrow 0} sup \frac{M - x_0}{\lambda} = \{v \in R^n : \lim_{\lambda \downarrow 0} inf \rho(v, \lambda^{-1}(M - x_0)) = 0\},$$

i.e. $v \in K_M(x)$ if for any $\alpha > 0$, $\varepsilon > 0$ there exists $u \in B_\varepsilon(0)$, $h \in (0,\alpha]$ such that $x + hu \in M$.

Definition 3. *[1]. A cone $K \subseteq K_M(x)$ is called a tent at $x \in M$, if there exists a mapping r, defined on some neighborhood U of zero, such that*

$$x + \bar{x} + r(\bar{x}) \in M, \text{ if } \bar{x} \in K \bigcap U \text{ and } \frac{r(\bar{x})}{\|\bar{x}\|} \to 0 \text{ as } \bar{x} \to 0.$$

The tent K is called continuous, if the map r is continuous.

Definition 4. *[2]. A mapping is called directionally differentiable at the point x_0, if for all $\bar{x} \in R^n$ the limit*

$$y'(x_0, \bar{x}) = \lim_{\lambda \downarrow 0} \frac{y(x_0 + \lambda \bar{x}) - y(x_0)}{\lambda}$$

exists.

Let a be a set-valued mapping and $(x_0, y_0) \in graf(a)$. Denote by $D^K(x_0, y_0)$ the set-valued mapping, the graph of which represents the cone $K \subseteq K_{graf(a)}(x_0, y_0)$.

Definition 5. *The set-valued mapping $D^K a(x_0, y_0)$ is called the derivative of a at the point $z_0 \equiv (x_0, y_0) \in graf(a)$.*

The following statement is valid.

Theorem 1. *Let $a : R^n \to 2^{R^m}$ be a set-valued mapping with starlike graph possessing a convex, closed sets of values. Suppose*

$$(x_0, y_0) \in graf(a^0), \quad x_0 \in int\, dom(a^0).$$

Then for any $(\hat{x}, \hat{y}) \in graf(D^{K_{graf(a)}}(x_0, y_0))$ there exists a directionally differentiable selection y of a defined in some neighborhood of x_0 and

$$y(x_0) = y_0, \ y'(x_0, \hat{x}) = \hat{y}, \ y(\bar{x}) \in D^{K_{graf(a)}}(x_0, y_0)(\bar{x}) \ \forall \bar{x} \in R^n.$$

Theorem 2. *Let $a : R^n \to 2^{R^m}$ be a set-valued mapping and let a convex closed cone $K \subseteq K_{graf(a)}(x_0, y_0)$ be a continuous tent to $graf(a)$ at the point (x_0, y_0). Suppose that $dom(D^K a(x_0, y_0)) = R^n$. Then for any $(\hat{x}, \hat{y}) \in K$ there exists a directionally differentiable selection y of a, defined in some neighborhood of x_0 such that*

$$y(x_0) = y_0 \text{ and } y'(x_0, \hat{x}) = \hat{y} \ \ y(\bar{x}) \in D^K a(x_0, y_0)(\bar{x}) \ \forall \bar{x} \in R^n.$$

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On the convergence of hard sampling operators

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We will consider the convergence of hard sampling operators with respect to the Haar double system. Let $\{h_n(x)\}_{n=1}^{\infty}$ be the Haar orthonormal system. For any $f(x,y) \in L_{[0,1]^2}$ denote

$$c_{\mathbf{n}}(f,\mathcal{H}) = c_{i,k}(f,\mathcal{H}) = \int\int_{[0,1]^2} f(x,y)h_i(x)h_k(y)dxdy.$$

The spectrum of $f(x,y)$ (denoted by $spec(f)$) is the index set where $c_{\mathbf{n}}(f)$ is non-zero: $spec(f) = \{\mathbf{n}, c_{\mathbf{n}}(f) \neq 0, \mathbf{n} = (i,k)\}$.

The operator

$$T_\lambda(f,x,y) = \sum_{|c_{i,k}|>\lambda} c_{i,k}(f)h_i(x)h_k(y),\ \lambda > 0$$

is called a hard sampling operator.

Let $\mathcal{B}$ be the class of sequences $\{b_{\mathbf{n}}\}$ such that

$$b_{\mathbf{n}} = 2^{-\frac{(m+l)}{2}}\hat{b}_m^{(1)}\hat{b}_l^{(2)},$$

where $\mathbf{n} = (i,k),\ i \in (2^m, 2^{m+1}], k \in (2^l, 2^{l+1}]$ and $\{\hat{b}_s^q\}_{s=0}^{\infty} \notin l^2, \hat{b}_s^{(q)} \searrow 0, q = 1,2.$

The following theorems are true for hard sampling operators:

Theorem 1. *Suppose that $\{b_{\boldsymbol{n}}\} \in \mathcal{B}$ and $\varepsilon > 0$. Then for each measurable function $f(x,y), (x,y) \in [0,1]^2$ there exists a bounded function $g(x,y), (x,y) \in [0,1]^2$, such that*

1. *$mes\{x,\ g(x,y) \neq f(x,y)\} < \varepsilon$,*
2. *$T_\lambda(g,x,y) \rightrightarrows g(x,y)$ as $\lambda \to 0$ (uniformly on $[0,1]^2$),*
3. *$c_{\boldsymbol{n}}(g) = b_{\boldsymbol{n}}$ for all $textbf n = (i,k) \in spec(g)$.*

Theorem 2. *Suppose that $\{b_{\boldsymbol{n}}\} \in \mathcal{B}$ and $\varepsilon > 0$. Then there exists a measurable set $E_\varepsilon \subset [0,1]^2$ with measure $mesE_\varepsilon > 1-\varepsilon$, such that for every function $f \in L^1_{E_\varepsilon}$ there is a function $g \in L^1_{[0,1]^2}$ coinciding with $f(x,y)$ on E_ε and satisfying the following statements:*

1. *$T_\lambda(g,x,y)$ converges to $g(x,y)$ a.e on $[0,1]^2$ and in $L^1_{[0,1]^2}$ norm as $\lambda \to 0$,*

2. *both the rectangular and the spherical partial sums of the double Fourier-Haar series of g converge to it almost everywhere,*
3. $c_{\boldsymbol{n}}(g) = b_{\boldsymbol{n}}$ *for all* $\boldsymbol{n} = (i,k) \in spec(g)$.

An interpolatory estimate for the UMD–valued directional Haar projection

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The Calculus of Variations, in particular the theory of compensated compactness has long been a source of hard problems in harmonic analysis. One development started with the work of F. Murat and L. Tartar, where the decisive theorems were on Fourier multipliers of Hörmander type. Using time–frequency localizations relying on Littlewood–Paley and wavelet expansions, as well as modern Calderon–Zygmund theory, S. Müller and J. Lee, P. F. X. Müller, S. Müller extended and strengthened the results obtained by Fourier multiplier methods.

In this paper we avoid this time–frequency localization by utilizing T. Figiel's canonical martingale decomposition instead, which permits us to further extend the results to the Bochner–Lebesgue space $L^p_X(\mathbb{R}^n)$, provided X satisfies the UMD–property. Given $n \geq 2$, $1 < p < \infty$ and $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n) \in \{0,1\}^n \setminus \{0\}$, define the directional Haar projection by $P^{(\varepsilon)}u = \sum_{Q \in \mathcal{Q}} \langle u, h^{(\varepsilon)}_Q \rangle \, h^{(\varepsilon)}_Q \, |Q|^{-1}$, for all $u \in L^p_X(\mathbb{R}^n)$. The Haar function $h^{(\varepsilon)}_Q$ is supported on the dyadic cube Q, has zero mean in each of the coordinates i whenever $\varepsilon_i = 1$, and $h^{(\varepsilon)}_Q(Q) = \{\pm 1\}$. Let R_i, $1 \leq i \leq n$ denote the Riesz-transform acting on the i–th coordinate. If $\varepsilon_{i_0} = 1$ and L^p_X has type $\mathcal{T}(L^p_X)$, the main result of this paper is the interpolatory estimate

$$\|P^{(\varepsilon)}u\|_{L^p_X} \leq C \, \|u\|_{L^p_X}^{1/\mathcal{T}(L^p_X)} \cdot \|R_{i_0}u\|_{L^p_X}^{1-1/\mathcal{T}(L^p_X)},$$

holding true for all $u \in L^p_X$, where $C > 0$ depends only on n, p, X and $\mathcal{T}(L^p_X)$.

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Equivalence of the absolute and unconditional convergences for wavelet systems

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It is known that the absolute and unconditional convergences are equivalent for numerical series, but for functional series this fact is not true in general. But there are functional series for which this equivalence remains true. For example, series by Haar system (see [3]), classical Franklin system (see [1]) or by general Franklin system ([2]) satisfy this property.

We consider similar problem regarding the wavelet systems.

Let f be a wavelet and $f_{jk}(x) = 2^{j/2} f(2^j x - k), \quad x \in \mathbb{R},\ j,k \in \mathbb{Z}$. Consider the following piecewise constant function $\varphi(x) = \int_m^{m+1} |f(t)| dt$, when $x \in [m, m+1)$, $m \in \mathbb{Z}$. We assume that there exist functions $\Phi_1(x)$ and $\Phi_2(x)$ and constants $C > 0$ and $0 < q < 1$ s.t.

$$\Phi_1(x) \leq \varphi(x) \leq \Phi_2(x),\ x \in \mathbb{R}, \tag{1}$$

$$\Phi_2(x) \leq C\Phi_1(x),\ x \in \mathbb{R}, \tag{2}$$

$$\Phi_2(\pm m) \leq q\Phi_2(\pm(m-1)),\ m \in \mathbb{N}, \tag{3}$$

$$\max_{x \in [m,m+1)} |f(x)| \leq C\varphi(m),\ m \in \mathbb{Z}. \tag{4}$$

Note that, for example, all wavelets with bounded support, Stromberg's polynomial wavelets satisfy the conditions above.

We prove that for wavelets satisfying (1)–(4) the following theorem is true

Theorem 1. *Let $a > 0$. Then the unconditional convergence a.e. on E, $\mu(E) > 0$ of series*

$$\sum_{\substack{j \in \mathbb{N},\ k \in \mathbb{Z} \\ \Delta_{jk} \subset (-a,a)}} a_{jk} f_{jk}(x)$$

is equivalent to its absolute convergence a.e. on E.

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On a representation of interpolating functions for biorthogonal systems of exponents

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Let $\{\lambda_k\}_{k=1}^n$ be a finite sequence of distinct complex numbers from $\Pi_+ = \{\lambda : \operatorname{Re}\lambda > 0\}$. For $0 < a \le +\infty$ denote by $\varphi_k^{(a)}(x)$ $(k = 1, 2, ..., n)$ the functions belonging to the linear span of the exponents $\left\{e^{-\lambda_k x}\right\}_{k=1}^n$ and satisfying the conditions $J_k^{(a)}(\lambda_r) = \delta_{kr}$ $(k, r = 1, 2, ..., n)$, where

$$J_k^{(a)}(\lambda) = \int_0^a \overline{\varphi_k^{(a)}(t)}\, e^{-\lambda t} dt, \quad \lambda \in \Pi_+ \text{ and } \delta_{kr} \text{ is the Kronecker's delta.}$$

Let a be fixed within $(0, +\infty]$. The problem is to examine the existence of a function $L(\lambda)$, analytic on Π_+, such that

$$\frac{L(\lambda)}{L'(\lambda_k)(\lambda - \lambda_k)} = J_k^{(a)}(\lambda),\ \lambda \in \Pi_+. \tag{1}$$

This form is useful to acquire $\varphi_k^{(a)}$ explicitly. For $a = +\infty$ the Blashke product $L(\lambda) = \prod_{k=1}^n \frac{\lambda-\lambda_k}{\lambda+\overline{\lambda_k}}$ fits (1) (see [1]). *However, for finite values of a such a function does no longer exist.*

Let us assume the opposite:

$$L(\lambda) = L'(\lambda_1)(\lambda - \lambda_1)J_1^{(a)}(\lambda) = L'(\lambda_2)(\lambda - \lambda_2)J_2^{(a)}(\lambda) = ...$$

$$... = L'(\lambda_n)(\lambda - \lambda_n)J_n^{(a)}(\lambda). \tag{2}$$

On the other hand, if $k \neq r$ then

$$\frac{J_r^{(a)}(\lambda)}{\lambda - \lambda_k} = \int_0^a \psi_r^{(a)}(t)\, e^{\lambda_k t} e^{-\lambda t} dt, \tag{3}$$

where $\psi_r^{(a)}(t) = \int_0^t \overline{\varphi_r^{(a)}(t)}\, e^{-\lambda_k t} dt$. From (2) and (3) one can deduce that the functions $\overline{\psi_r^{(a)}(t)}\, e^{\overline{\lambda_k} t}$ are generated by $\left\{e^{-\lambda_k x}\right\}_{k=1}^n$ and conclude that

$$\int_0^\infty \overline{\varphi_r^{(a)}(t)}\, e^{-\lambda_k t} dt = 0 \quad (k \neq r).$$

The uniqueness of generated biorthogonal systems (GBS) implies $\varphi_r^{(a)}(t) = c_r^{(a)} \varphi_r^{(+\infty)}(t)$ $(c_r^{(a)} \in \mathbb{C} \setminus \{0\})$. This is why

$$\int_0^a \overline{\varphi_r^{(+\infty)}(t)}\, e^{-\lambda_k t} dt = 0 \quad (k \neq r,\ 0 < a < \infty),$$

therefore, $\varphi_r^{(+\infty)}(t) \equiv 0$, which is false.

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Analytically nonextendable multidimensional power series

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It is well known that analytically nonextendable power series are easy to construct in the class of so-called *strong lacunar* series, in other words, when there are "many" monomials with zero coefficients.

In 1891, Fredholm [1] gave examples of *moderate lacunar* nonextendable series, moreover, these series were represented as infinitely differentiable functions in the closure of the convergence disk. These series depend on a parameter a, and they have the following form

$$\sum_{n=0}^{\infty} a^n z^{n^2}, \qquad 0 < |a| < 1.$$

The theorem 1 demonstrates that in Fredholm's example the power order of lacunarity can be improved from 2 to $1+\varepsilon$ and can be generalized to multidimensional power series.

Theorem 1. *If the increasing sequences of natural numbers n_{k_i} satisfies the inequality $n_{k_i} \geq const \cdot {k_i}^{1+\varepsilon}$, $i = 1, ..., m$, with $\varepsilon > 0$, then the power series*

$$\sum_{k \in N^m} a^k z^{n_k}, \qquad 0 < |a_i| < 1 \tag{1}$$

is not extendable across the boundary of the convergence domain, but represents an infinitely differentiable function in the closed domain.

Here $z^{n_k} = {z_1}^{n_{k_1}} {z_2}^{n_{k_2}} ... {z_m}^{n_{k_m}}$ and $a^k = {a_1}^{k_1} {a_2}^{k_2} ... {a_m}^{k_m}$, $m \in N$.

We use the Dirichlet series [2] in the proof of this Theorem.

In multidimensional cases, the interest of studying series of form (1) arises in the thermodynamics of several Hamiltonians [3].

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Pointwise estimates for spline Gram matrix inverses

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We give a proof of the pointwise matrix inequality of exponential decay

$$|b_{i,j}| \leq \frac{Cq^{j-i}}{t_{j+k} - t_i}, \qquad i \leq j. \tag{1}$$

Here, $(b_{i,j})_{i,j=1}^M$ is the inverse of the Gram matrix of B-spline functions of order k corresponding to the arbitrary point sequence $\mathcal{T} = (t_1 = \cdots = t_k < t_{k+1} < \cdots < t_M < t_{M+1} = \cdots = t_{M+k-1})$. The constants C and $q < 1$ depend only on the spline order k.

We illustrate the new method of proof to obtain (1) in the known case of piecewise linear splines ($k = 2$), but we were also able to extend our argument to show (1) for piecewise quadratic splines ($k = 3$).

The main focus lies on the method of proof that uses the Sherman-Morrison formula for inverses of matrices and subsequently induction on the size of the matrices. In particular, this approach does *not* use the celebrated result of A. Shadrin (2001) that for every spline of order k, the orthogonal spline projection operator $P_{\mathcal{T}}^k$ is bounded on $C[0,1]$ independently of $\mathcal{T}$. The result of Shadrin and (1) are related, since the uniform boundedness of $P_{\mathcal{T}}^k$ on $C[0,1]$ is equivalent to the inequality

$$|b_{i,j}| \le \frac{Cq^{j-i}}{\max(t_{i+k}-t_i, t_{j+k}-t_j)}, \qquad i \le j.$$

The inequality (1) can be used to show a.e. convergence and unconditional convergence in reflexive L^p spaces of orthogonal spline series corresponding to arbitrary points.

Duality in some spaces of functions harmonic in the unit ball

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A positive continuous decreasing function φ on $[0,1)$ is called a weight function, if $\lim \varphi(r) = 0$ as $r \to 1$, and a positive finite Borel measure η on $[0,1)$ is called a weighting measure, if it is not supported in any subinterval $[0,\rho)$, $0 < \rho < 1$. Let $h_\infty(\varphi)$ be the Banach space of functions u, harmonic in the unit ball $B_n \subset \mathbb{R}^n$, with the norm $\|u\|_\varphi = \sup\{|u(z)|\varphi(|z|)\colon\ |z| < 1\}$ and let $h_0(\varphi)$ be its closed subspace of functions u with $|u(z)| = o\big(1/\varphi(|z|\big)$ as $|z| \to 1$.

It has been shown by Rubel and Shields [1] that $h_\infty(\varphi)$ is isometrically isomorphic to the second dual of $h_0(\varphi)$. In [2] the duality problem of finding a weighting measure η such that

$$h^1(\eta) = \big\{v \in L^1(d\eta(r)\,d\theta)\colon\ v \text{ harmonic for } |z| < 1\big\}$$

represents the intermediate space, the dual of $h_0(\varphi)$ and the predual of $h_\infty(\varphi)$, i.e. $h^1(\eta) \sim h_0(\varphi)^*$ and $h^1(\eta)^* \sim h_\infty(\eta)$, was solved. In this duality relations the pairing is given by

$$\langle u, v\rangle = \frac{1}{2\pi}\int_0^{2\pi}\int_0^1 u(re^{i\theta})\overline{v(re^{i\theta})}\varphi(r)\,d\eta(r)\,d\theta, \quad u \in h_\infty(\varphi),\ \ v \in h^1(\eta).$$

In the indicated articles [1] and [2] the authors consider only the case $n = 2$, when harmonic function is a real part of the function, holomorphic in B_2. Therefore u has an expansion in a series on degrees of z and $\overline{z}$. This allows to apply the methods of complex analysis.

In the present work we consider the duality problem in the case of harmonic functions in the unit ball of $\mathbb{R}^n$, $n > 2$. The above approach cannot be applied in the multidimensional case, since we can not speak about connections between harmonic and holomorphic functions, and instead of degrees of z and $\overline{z}$ we deal with spherical harmonics.

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Convergence acceleration of the quasi-periodic interpolation by rational and polynomial corrections

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We consider the problem of function interpolation by the quasi-periodic trigonometric interpolations $I_{N,m}(f,x)$, $m \geq 0$ (m is an integer), $x \in [-1,1]$, which interpolate f on equidistant grid

$$x_k = \frac{k}{N}, \ |k| \leq N \tag{1}$$

and are exact for a quasi-periodic functions

$$e^{i\pi n\sigma x}, \ |n| \leq N, \ \sigma = \frac{2N}{2N+m+1} \tag{2}$$

with the period $2/\sigma$ (which tends to 2 as $N \to \infty$).

The quasi-periodic interpolations were suggested in [1]. Papers [2] and [3] investigated the L_2-convergence of the quasi-periodic interpolations on the entire interval and their behavior at the endpoints of the interval in terms of the limit function. Some results concerning these investigations were reported also in [4] and [5].

Here we study the pointwise convergence of the quasi-periodic interpolations in the regions away from the endpoints ($x = \pm 1$) and derive the exact constants of the asymptotic errors showing fast convergence compared to the classical interpolation.

Then we consider the convergence acceleration of the quasi-periodic interpolations by application of rational (by $e^{i\pi x}$) or polynomial corrections. The application of the corrections leads to rational-trigonometric, polynomial-trigonometric or rational-trigonometric-polynomial quasi-periodic interpolations. Rational corrections contain some unknown parameters which determination is important for realization of the corresponding interpolations. Polynomial corrections represent the values and some first derivatives of function at the endpoints of the interval. We study the convergence of the corresponding interpolations and consider the problem of parameter determination. We also consider the problem of approximation of derivatives based on the discrete Fourier coefficients.

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On multiple Fourier series of functions of bounded partial generalized variation

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The convergence of partial sums and Cesaro means of multiple Fourier series of bounded partial Λ-variation functions is investigated. The sufficient and necessary conditions on the sequence $\Lambda = \{\lambda_n\}$ are found for the convergence, as well as for the Cesaro summation of Fourier series of bounded partial Λ-variation functions.

Similar results are obtained for the multiple Fourier series with respect to the Walsh system.

This is a joint work with Ushangi Goginava.

Hermite type quadrature and cubature formulas

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In 1814 Carl Friedrich Gauss published his famous quadrature formula of the highest algebraic degree of accuracy. The Gaussian formula opened the central direction in numerical analysis devoted to the construction of approximation methods of a given form that are exact for all polynomials of the highest possible degree. The Gaussian setting reflected the classical understanding that regards the algebraic polynomials as simple and nice functions which can easily be differentiated, integrated, and evaluated at any point. Moreover, by the Weierstrass theorem, an arbitrary continuous function can be approximated by polynomials with arbitrary accuracy. For this reason, every approximation method which is good for a broad class of polynomials was regarded as a good method. Starting from the very appearance of the first computers, one can see an active consideration of theoretical problems related to the solution of extremal problems on classes of functions. At that time Andrei Nikolaevich Kolmogorov posed the following problem in the 1940s: *Construct a quadrature formula of a given type that has minimal error on a given class of functions.* But the optimal quadrature formulas may not be unique. More information about this can be found in a review article of B. Boyanov, "Optimal quadrature formulae", *Russian Math.Surveys*, 60:6, 2005 pp.1035-1055.

The main part of the report is devoted to the quadrature and cubature formulas of Hermitian type with the highest trigonometric degree of accuracy. The report provides some quadrature and cubature formulas of Hermitian type, obtained by the author. We will present our study on the convergence of the obtained quadrature formulas, their error estimates and numerical analysis, and the comparison with some well-known formulas.

On the divergence of Fourier-Walsh series

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It's proved that for any perfect set P and for any density point x_0 of P one can define a finite and measurable function $f(x)$ having the following properties: any finite and measurable function $g(x)$, coinciding with $f(x)$ on P, has a divergent Fourier-Walsh series at the point x_0.

Solving deconvolution problems by wavelet decomposition methods

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We consider the inverse problem associated to an equation of the form $Af = g$ where A is the convolution-type operator

$$Af(x) = \int h(w) e^{iwx} dx \hat{f}(w),$$

$h \in L^2_{loc}(\mathbb{R})$ is a smooth function and $f \in L^2(\mathbb{R})$. It consists in finding a solution f for given data g. When the data g is perturbed and the available data $\tilde{g}$ satisfies $\|g - \tilde{g}\| < \delta$, the IP may be ill-posed and small perturbations in the data can produce large errors in the solution. Then, approximated solutions $\overline{f}$ to the IP should be computed carefully, applying suitable regularization techniques.

Theoretical and numerical aspects of inverse problems associated to convolution-type operators with noisy discrete data have been studied in [5], [6], and [2] among others. In our case, if $h = \check{m}$ is a C^∞ function, A is a pseudodifferential operator with symbol $p(x, w) = m(w)$ (see [9]). Inverse Problems associated to this kind of operators have been studied in [3], [1], [4] and [8], where techniques based on Wavelet Galekin Methods and Wavelet Vaguelet Decomposition were developed. In this work, we construct the *elementary solutions*, smooth functions μ_{jk} that are *nearly* the preimages of a wavelet basis $\psi_{jk}(x) = 2^{j/2}\psi(2^j x - k)$ developed in [7]. The mother wavelet is smooth and infinitely oscillating function with fast decay and compact support in two-sided bands. It is well localized in both time and frequency domain and it is well adapted to this problem. We construct an approximated solution to the IP based on the decomposition of the perturbed data into the images of μ_{jk} via the operator A. Under certain hypothesis we estimate the error of the aproximation and discuss the advantages of the proposed scheme.

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On density of smooth functions in weighted Sobolev-Orlicz spaces

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In this work we are concerned with the question of density of smooth functions in weighted Sobolev-Orlicz spaces. In a bounded Lipshitz domain $\Omega \subset \mathbb{R}^n$ for a weight (nonnegative measurable function) ρ, satisfying

$$\rho,\ \rho^{-1} \in L^1(\Omega)$$

we define the Sobolev space W as the closure of the set of functions

$$X := \left\{ u \in W_0^{1,1}(\Omega) \, : \, \int_\Omega |\nabla u|^p \rho \,\mathrm{d}x < \infty \right\},$$

where $p = \mathtt{const} > 1$ with respect to the norm

$$\|u\|_W = \left(\int_\Omega |\nabla u|^p \rho \,\mathrm{d}x \right)^{1/p}.$$

We assume that the weight additionally satisfies $\rho^{1/(1-p)} \in L^1(\Omega)$, which guarantees completeness of X and $W \subset W_0^{1,1}(\Omega)$. Next, we can define another space, H, as the closure of C_0^∞ in W.

The classical question in the theory of Sobolev spaces is whether $H = W$. If $\Omega = \mathbb{R}^n$ and $\rho \equiv 1$ the question is simple and solved by mollifications. For the standard $W^{1,2}(\Omega)$ the question was solved in the famous paper of Meyers and Serrin entitled $H = W$. They did not require any smoothness of the boundary of Ω. If one makes some additional assumptions on the smoothness of $\partial\Omega$, it is possible to prove that functions smooth up to the boundary are also dense in the Sobolev space.

In the presence of a weight, though, the question naturally becomes more complicated. If one tries to follows the classical route, the uniform boundedness of mollifying operators is required. This question is closely related to the boundedness of the Hardy-Littlewood maximal function in the corresponding weighted Lebesgue space. For instance, this is guaranteed by ρ belonging to the Muckenhoupt class A_p. However, the Muckenhoupt condition is not easy to verify, and, second, the story does not end here.

Another approach to approximating a Sobolev function with smooth functions is the so called *Lipschitz truncation* technique, based on cutting the original function at some level of the maximal function of its gradient and using the McShane extension theorem. This technique was successfully applied by a number of authors for the study of very weak solutions of elliptic PDEs, in mathematical hydrodynamics and so on. In this work we use this technique to establish the following sufficient condition for density of smooth functions in weighted Sobolev spaces.

Theorem 1. *Let $\rho = ww_0$, $w_0 \in A_p$. Assume that*

$$\liminf_{t\to\infty} \frac{\|w\|_{L^t(\Omega\,;\,w_0\mathrm{d}x)} \cdot \|w^{-1}\|^{1-\varepsilon(t)}_{L^t(\Omega\,;\,w_0\mathrm{d}x)}}{t^p} < \infty, \quad \textit{where} \quad \varepsilon(t) = \frac{2}{t+1}.$$

Then $H = W$.

Next, we extend this result to the case of Sobolev spaces with the variable exponent $p = p(x)$. We assume that the exponent satisfies

$$1 < \alpha < p(x) < \beta < \infty, \tag{1}$$

$$|p(x) - p(y)| \le \frac{k_0}{\ln \frac{1}{|x-y|}}, \quad |x-y| < 1. \tag{2}$$

This is the widely known logarithmic condition introduced originally by V.V. Zhikov and X.L. Fan. The definition of H and W is the same with the difference that the Lebesgue norm $\|\cdot\|_W$ introduced above is replaced by the Luxemburg norm. If $\rho = 1$ it guarantees the absence of the Lavrentiev phenomenon which is equivalent to $H = W$.

Theorem 2. *Let*

$$\liminf_{t\to\infty} \|\rho\|_{L^t(\Omega\,;\,\mathrm{d}x)} \cdot \|t^{-p}\rho^{-1}\|^{1-\varepsilon(t)}_{L^t(\Omega\,;\,\mathrm{d}x)} < \infty, \quad \textit{where} \quad \varepsilon(t) = \frac{2}{t+1}.$$

Then $H = W$.

It is easily seen that in the case when $p = \mathtt{const}$ this theorem reduces to the first one. However, the question whether one can take ρ as in the constant exponent case remains open. The definition of the Muckenhoupt classes for variable exponents and the corresponding toolbox is still an open problem, though recently serious progress has been achieved by L. Diening and P. Hästö. Another very interesting question is whether the logarithmic condition (2) can be weakened.

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On partial sums and Fejer means of Walsh-Fourier series

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The main aim of my talk will be to investigate partial sums and Fejer means of Walsh-Fourier series in the Hardy spaces. We will present necessary and sufficient conditions for positive numbers such that the partial sums and Fejer means with this indices are bounded in the Hardy spaces, when $0 < p < 1$.

Lower bounds for L_1 discrepancy

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We give the best asymptotic lower estimate for the coefficient of the leading term for the L_1 norm of the two-dimensional axis-parallel discrepancy that could be obtained by K.Roth's orthogonal function method in a large class of test functions.

The proof is based on the methods of combinatorics, probability, complex and harmonic analysis.

Generalized convex sets in the hypercomplex space

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The report will focus on analogs for hypercomplex case of some results in complex linear convex analysis, which was studied by one of the authors [1]. Significant differences of the generalized convexity (global and local) in the complex and hypercomplex cases will be analyzed and the main difficulties arising from the non-commutativity of the quaternion field will be discussed. We will give an overview of currently known results, focus on open problems and perspectives of their solutions [1-2].

Theorem 1. *([3]) A compact, which has the form of a non-degenerate Cartesian product in $\mathbb{N}^n$, strongly linearly convex if and only if it is convex.*

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Differential and Integral Equations

Homogenization of Dirichlet problem for divergence type elliptic systems

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Let D be a bounded domain in $\mathbb{R}^d$ $(d \geq 2)$, $A(y) = (A_{ij}^{\alpha\beta}(y))$, $1 \leq \alpha, \beta \leq d$, $1 \leq i, j \leq N$ be an $\mathbb{R}^{N^2 \times d^2}$-valued function defined on $\mathbb{R}^d$, and g be a $\mathbb{C}^N$-valued function defined on $\overline{D} \times \mathbb{T}^d$. We study the asymptotic behavior of solutions to the following problem:

$$\mathcal{L} u_\varepsilon(x) = 0, \text{ in } D, \text{ and } u_\varepsilon(x) = g(x, x/\varepsilon), \text{ on } \partial D,$$

where $\varepsilon > 0$ is a small parameter, and using the summation convention of repeated indices the operator $\mathcal{L}$ is defined as

$$(\mathcal{L}u)_i := -\frac{\partial}{\partial x^\alpha}\left[A_{ij}^{\alpha\beta}(x)\frac{\partial u_j}{\partial x^\beta}\right] = -\mathrm{div}\left[A(x)\nabla u\right],$$

where $u = (u_1, ..., u_N)$ and $1 \leq i \leq N$. We make the following assumptions:

(i) (Periodicity) The boundary function g is 1-periodic:

$$g(x, y+h) = g(x,y), \ \forall x \in D, \forall y \in \mathbb{R}^d, \ \forall h \in \mathbb{Z}^d.$$

(ii) (Ellipticity) There exists a constant $c > 0$ such that

$$c^{-1}\xi_\alpha^i \xi_\alpha^i \leq A_{ij}^{\alpha\beta}(y)\xi_\alpha^i \xi_\beta^j \leq c\xi_\alpha^i \xi_\alpha^i, \qquad \forall \xi \in \mathbb{R}^{d \times N}.$$

(iii) (Smoothness) The boundary value g and all elements of A are sufficiently smooth.

For smooth and uniformly convex domains we study the pointwise, and L^p, $1 \leq p < \infty$, limit behavior of solutions u_ε, as $\varepsilon \to 0$. In non smooth setting we study the problem in case of equations $(N = 1)$ set in convex polygonal domains, i.e. domains that are bounded by a finite number of hyperplanes. In this case we show that the limit behavior of solutions is intimately connected with certain number-theoretic properties of the normal vectors of the bounding hyperplanes. We will also discuss some further applications of our methods when the coefficients of the operator $\mathcal{L}$ are 1-periodic and have oscillations in ε-scale too.

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Numerical approximation for a variational problem of the spatial segregation of Reaction-Diffusion system

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Recently, much interest has gained the numerical approximation of equations of the Spatial Segregation of Reaction-diffusion systems with disjoint population densities. These problems are governed by a minimization problem subject to *closed* but *non-convex* set.

In this talk I will present a numerical approximation of equations of stationary states for a certain class of the Spatial Segregation of Reaction-diffusion system with m population densities having disjoint support. We use quantitative properties of both solutions and free boundaries to derive our scheme. We prove that the developed scheme is stable and consistent. Moreover, the proof of convergence of the proposed numerical method is given in some particular cases. At the end of the talk I will show and discuss the numerical implementations of the resulting approach.

On the Dirichlet problem for the fourth order partial differential equation when characteristic equation has only double roots

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Let D be the unit disk of the complex plane with boundary $\Gamma = \partial D$. We consider in D the following equation

$$\left(\frac{\partial}{\partial y} - \lambda_1 \frac{\partial}{\partial x}\right)^2 \left(\frac{\partial}{\partial y} - \lambda_2 \frac{\partial}{\partial x}\right)^2 u = 0, \tag{1}$$

where λ_j $(j = 0, 1)$ are complex constants, such that $\lambda_1 \neq \lambda_2$. We seek the solution u of the equation (1) from the class $C^4(D) \bigcap C^{(1,\alpha)}(D \bigcup \Gamma)$ satisfying on the boundary Γ the Dirichlet boundary conditions

$$\left.\frac{\partial^k u}{\partial r^k}\right|_\Gamma = f_k(x, y), \quad k = 0, 1, \quad (x, y) \in \Gamma. \tag{2}$$

Here f_k $(k = 0, 1)$ are prescribed functions on Γ, $\frac{\partial}{\partial r}$ is the derivative with respect to module of the complex number $(z = re^{i\varphi})$.

If $\Im\lambda_1 > 0 > \Im\lambda_2$ (the equation (1) is properly elliptic), we suppose, that $f_k \in C^{(1-k,\alpha)}(\Gamma)$. In this formulation the problem (1), (2) is Fredholm, (see [1]), and in the paper [2] the necessary and sufficient conditions of the unique solvability of this problem, and formulas to calculate the defect numbers of the problem (i.e. the number of linearly independent solutions of the homogeneous problem (1), (2), when $f \equiv 0$, and the number of linearly independent solvability conditions of the inhomogeneous problem), if the unique solvability fails, were found. This formulation of the Dirichlet problem is no longer correct in the case $\Im\lambda_1 \geq \Im\lambda_2 > 0$ (improperly elliptic equation (1))([3]), therefore in this talk we change the class of boundary functions in the following way.

Let $\Im\lambda_1 \geq \Im\lambda_2 > 0$. We denote $\mu = \frac{i-\lambda_1}{i+\lambda_1}$, $\nu = \frac{i-\lambda_2}{i+\lambda_2}$, and assuming $|\mu| \leq |\nu|$, denote by $B^{(m,\alpha)}(|\nu|)$ the set of functions, analytic in the ring $|\nu| < |z| < 1$ and Hölder continuous with k-th order derivatives $(k = 1, \ldots, m)$ in the closed ring $|\nu| \leq |z| \leq 1$. We suppose that $f_k \in B^{(2-k,\alpha)}(|\nu|)$, $k = 0, 1$.

We prove that in this formulation the problem (1), (2) is Fredholm, and find the formulas for the defect numbers of the problem (different from the formulas appearing in [2]). These formulas are the same in the cases of properly and improperly equations (1). The case of real λ_j (non-elliptic equation (1)) is considered too.

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On differential invariants and contact equivalence of ordinary differential equations

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The problem of classification of ordinary differential equations with respect to the contact or point transformations is one of the most important problems in the theory of differential equations. It seems that the most interesting one is the case of differential equations of type

$$y^{(n)} = f(x, y, y', \dots, y^{(n-1)}). \qquad (1)$$

The first results in this area were obtained by Sophus Lie:

1) *ordinary differential equation (1) of the first order in neighbourhood of regular point is point–equivalent to the equation* $y' = 0$;

2) *ordinary differential equation (1) of the second order in neighbourhood of regular point is contact–equivalent to the equation* $y'' = 0$.

The point classification of regular equations (1) of order 2 was obtained by Tresse (see [2, 4]). Non–regular equations were studied by Lie, Tresse, Liuville, Cartan, etc. (see, for example, [5]).

Contact equivalence problem for differential equations (1) of order $n \geqslant 3$ was studied by Cartan, Chern, Fels, etc. Finally, Doubrov, Komrakov and Morimoto constructed the Cartan connection associated with every differential equation (1) (and, more generally, with every holonomic system of differential equations), and reduced the contact equivalence problem to the classical problem of equivalence of $\{e\}$-structures (see [3]). So, the equivalence problem of ordinary differential equations (1) can be solved in terms of $\{e\}$-structures.

Nevertheless, another important problem in this area was still open. Namely, *how can the differential invariant field of the action of contact pseudogroup on differential equations (1) be described*?

In this work we solve this problem for differential equations of arbitrary order $n \geqslant 3$. Also in terms of this field we obtain a new approach to contact classification of ordinary differential equations.

Theorem 1. *Differential invariant field of the action of contact pseudogroup on differential equations is generated by arbitrary differential invariant J and two special invariant operators δ and Δ.*

If we restrict these invariants on given differential equation $\mathcal{E}$, one can write

$$\delta^{n+1}J = \mathcal{P}_{\mathcal{E}}(J, \delta J, \ldots, \delta^n J), \qquad \Delta J = \mathcal{Q}_{\mathcal{E}}(J, \delta J, \ldots, \delta^n J)$$

for some smooth functions $\mathcal{P}_{\mathcal{E}}$ and $\mathcal{Q}_{\mathcal{E}}$.

Theorem 2. *Two differential equations $\mathcal{E}$ and $\widetilde{\mathcal{E}}$ are contact equivalent iff $(\mathcal{P}_{\mathcal{E}}, \mathcal{Q}_{\mathcal{E}}) = (\mathcal{P}_{\widetilde{\mathcal{E}}}, \mathcal{Q}_{\widetilde{\mathcal{E}}})$.*

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On solvability of Dirichlet problem for general second order elliptic equation

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In a bounded domain $Q \subset R_n$, $n \geq 2$, with C^1-smooth boundary ∂Q, we study the solvability in the $W^1_{2,\,loc}(Q)$-setting of the Dirichlet problem for the second-order elliptic equation

$$-div\,(A(x)\nabla u) + (\bar{b}(x), \nabla u) - div(\bar{c}(x)u) + d(x)u = f(x) - divF(x), \quad x \in Q, \tag{1}$$

$$u|_{\partial Q} = u_0, \tag{2}$$

with boundary function u_0 in $L_2(\partial Q)$. We assume that f and $F = (f_1, \dots, f_n)$ are in $L_{2,loc}(Q)$ and that the symmetric matrix $A(x){=}(a_{ij}(x))$, consisting of real-valued measurable functions, satisfies the condition

$$\gamma|\xi|^2 \le \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j = (\xi, A(x)\xi) \le \gamma^{-1}|\xi|^2$$

for all $\xi = (\xi_1, \dots, \xi_n) \in R_n$ and almost all $x \in Q$, where γ is a positive constant. We also assume that the coefficients $\bar{b}(x) = (b_1(x), \dots, b_n(x))$, $\bar{c}(x) = (c_1(x), \dots, c_n(x))$ and $d(x)$ are measurable and bounded on any strictly interior subdomain of Q.

It was shown that if a $W^1_{2,loc}(Q)$-solution of the Dirichlet problem (1), (2) exists, it is $(n-1)$-dimensionally continuous (i.e., belongs to the Gushchin space $C_{n-1}(\bar{Q})$); this characterizes behavior of the solution near the boundary, and describes in what sense it takes its boundary value. This was done under the assumption that the lower order coefficients $\bar{b}(x)$, $\bar{c}(x)$ and $d(x)$ are locally bounded and satisfy the following conditions:
there exists a constant $M > 0$ such that

$$|\bar{b}(x)| \le \frac{M}{r(x)(1 + |\ln r(x)|)^{3/4}}, \quad x \in Q,$$

$$\int_0 t(1 + |\ln t|)^{3/2} C^2(t)dt < \infty, \quad \text{where } C(t) \equiv \sup_{r(x) \ge t} |\bar{c}(x)|,$$

$$\int_0 t^3(1 + |\ln t|)^{3/2} D^2(t)dt < \infty, \quad \text{where } D(t) \equiv \sup_{r(x) \ge t} |d(x)|.$$

Here $r(x)$ is the distance from a point $x \in Q$ to the boundary ∂Q.

Conditions for the existence of an $(n-1)$-dimensionally continuous solution are obtained, the resulting solvability condition is shown to be similar in form to the solvability condition in the conventional generalized setting (in $W^1_2(Q)$). In particular, it is proven that if the homogeneous problem (with zero boundary conditions and zero right-hand side) has no nonzero solution in $W^1_2(Q)$, then for all $u_0 \in L_2(\partial Q)$ and all f and F from the appropriate function spaces there exists a solution for the non-homogeneous problem (in $W^1_{2,\,loc}(Q)$-setting); this solution belongs to Gushchin space $C_{n-1}(\bar{Q})$ of $(n-1)$-dimensional continuous functions and the following estimate holds

$$\|u\|^2_{C_{n-1}(\bar{Q})} + \int_Q r\,|\nabla u|^2 dx \le$$

$$\leq C\Big(\|u_0\|^2_{L_2(\partial Q)} + \int\limits_Q r^3 \left(1+|\ln r|\right)^{\frac{3}{2}} f^2 dx + \int\limits_Q r \left(1+|\ln r|\right)^{\frac{3}{2}} |F|^2 dx\Big),$$

where the constant is independent of u_0, f, F.

For some natural constraints on the lower order coefficients of the equation (1):

$$\int\limits_0 B^2(t)dt < \infty, \quad \int\limits_0 C^2(t)dt < \infty, \quad \int\limits_0 tD^2(t)dt < \infty,$$

where $B(t) \equiv \sup\limits_{r(x)\geq t} |\bar{b}(x)|$, for the right-hand sides from $W_2^{-1}(Q)$ the obtained necessary and sufficient conditions for solvability of the problem (1),(2) in $W^1_{2,loc}(Q)$-setting can be stated in a simpler form - in terms of the original problem. Also it is proven that if the boundary function has an extension to Q that belongs to $W_2^1(Q)$, then the solution in $C_{n-1}(\bar{Q})$ is at the same time a solution in $W_2^1(Q)$ and in such a case the solvability conditions of the problem (1),(2) in $W^1_{2,loc}(Q)$-setting coincide with those in $W_2^1(Q)$-setting.

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On manifolds of periodic eigenvalue problems with selected double eigenvalue

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We consider the family

$$-y'' + p(x)y = \lambda y, \quad y(0) - y(2\pi) = y'(0) - y'(2\pi) = 0$$

of periodic eigenvalue problems with 2π-periodic real potential

$$p \in P := \left\{ C(0, 2\pi) \mid \int_0^{2\pi} p(x)dx = 0 \right\}$$

as a functional parameter. For a fixed potential p, the spectrum of the problem consists of isolated real eigenvalues, the multiplicity of the eigenvalues does not exceed two, and the spectrum has the form

$$\lambda_0(p) < \lambda_1^-(p) \leq \lambda_1^+(p) < \ldots < \lambda_k^-(p) \leq \lambda_k^+(p) < \ldots \to \infty.$$

Our interest is the subsets

$$P_k := \{p \mid \lambda_k^-(p) = \lambda_k^+(p)\} \subset P \quad k = 1, 2, ...$$

of potentials with a double k-eigenvalue. We give a new description of the topological structure of the subset P_k and its complement $P \setminus P_k$ (another approach to the problem is described in [1]). In particular, we prove the Arnold "hypothesis of transversality" (V. I. Arnold, Modes and quasimodes. Functional Analysis and its Applications 6 (1972), no. 2, 12–20) that the subset P_k is a smooth submanifold of codimension two. Then, we show that the manifold P_k is homotopically trivial, the complement $P \setminus P_k$ is trivially fibered over P_k and the standard fiber is the product $\mathbb{R}P^1 \times \mathbb{R}^+$, i.e.,

$$P \setminus P_k \cong P_k \times (\mathbb{R}P^1 \times \mathbb{R}^+).$$

Finally, we prove that the linking coefficient $L(k,p)$ of the submanifold P_k with a loop of shifts

$$l(p,k) := \{p(x+t) \in P \setminus P_k \mid t \in [0, 2\pi]\}$$

is $L(k,p) = 2k$.

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The density of the smooth compactly supported functions in a weighted multi - anisotropic Sobolev space

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Let $1 < p < \infty$, $\delta > 0$, $g_\delta(x) = e^{-\delta|x|}$ and let $\Re$ be a Newton polyhedron. Denote by $W^{\Re}_{p,\,\delta}$ the set of locally integrable functions in E^n with bounded norms

$$||u||_{W^{\Re}_{p,\,\delta}} \equiv \sum_{\alpha\in\,\Re} ||(D^\alpha u)g_\delta||_{L_p} = \sum_{\alpha\in\,\Re} ||D^\alpha u||_{L_{p,\,\delta}}.$$

It is proved that $W^{\Re}_{p,\,\delta}$ is a Banach space (see [1]) . Such Sobolev type weighted spaces arise in connection with numerous problems in the theory of partially hypoelliptic (see [2]) and almost hypoelliptic (see [3]) differential equations. We prove the following result:

Theorem 1. *Let $\Re$ be a completely regular polyhedron (see [3]). The space $W^{\Re}_{p,\,\delta}$ is semilocal (see [2]) and the set $C_0^\infty(E^n)$ is dense in $W^{\Re}_{p,\,\delta}$.*

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The inverse problem for a family of Sturm-Liouville operators

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Let us denote by $\mu_n(q,\alpha,\beta)$, $n = 0,1,2,\dots$ the eigenvalues of the Sturm-Liouville problem $L(q,\alpha,\beta)$:

$$-y'' + q(x)\,y = \mu y,\ x \in (0,\pi)\,,\ q \in L^1_R[0,\pi]\,,\ \mu \in C, \tag{1}$$

$$y(0)\cos\alpha + y'(0)\sin\alpha = 0,\ \alpha \in (0,\pi]\,, \tag{2}$$

$$y(\pi)\cos\beta + y'(\pi)\sin\beta = 0,\ \beta \in [0,\pi). \tag{3}$$

By $\varphi(x,\mu,\alpha)$ we denote the solution of (1), which satisfies the initial conditions $\varphi(0,\mu,\alpha) = \sin\alpha, \varphi'(0,\mu,\alpha) = -\cos\alpha$. It is easy to see that $\varphi_n(x) \overset{def}{=} \varphi(x,\mu_n,\alpha)$, $n = 0,1,2,\ldots$ are the eigenfunctions of $L(q,\alpha,\beta)$. The quantities $a_n(q,\alpha,\beta) = \int\limits_0^\pi |\varphi_n(x)|^2 dx$, $n = 0,1,2,\ldots$ are called the norming constants.

The sequences $\{\mu_n\}_{n=0}^\infty$ and $\{a_n\}_{n=0}^\infty$ are called the spectral data of the problem $L(q,\alpha,\beta)$.

The following result is well known (see [1]):

For the sequences $\{\mu_n\}_{n=0}^\infty$ and $\{a_n\}_{n=0}^\infty$ to be the spectral data for a **certain** problem $L(q,\alpha,\beta)$, with $q \in L_R^2[0,\pi]$, it is necessary and sufficient that $\{\mu_n\}_{n=0}^\infty$ and $\{a_n\}_{n=0}^\infty$ satisfy some relations (see [1]).

What additional relations must be satisfied for $\{\mu_n\}_{n=0}^\infty$ and $\{a_n\}_{n=0}^\infty$ to be the spectral data of the problem $L(q,\alpha,\beta)$ with fixed α and β?

We answer this question.

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On the "movement" of the zeros of eigenfunctions of the Sturm-Liouville problem

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Let us consider the Sturm-Liouville problem

$$-y'' + q(x)y = \mu y,\ x \in (0,\pi),\ q \in L_R^1(0,\pi),\ \mu \in C, \tag{1}$$

$$y(0)\cos\alpha + y'(0)\sin\alpha = 0,\ \alpha \in (0,\pi], \tag{2}$$

$$y(\pi)\cos\beta + y'(\pi)\sin\beta = 0,\ \beta \in [0,\pi), \tag{3}$$

and let $\mu_n(q,\alpha,\beta)$, $n = 0,1,2,\ldots$ be the eigenvalues, and $y_n(x) = y_n(x,q,\alpha,\beta)$, $n = 0,1,2,\ldots$ be the corresponding eigenfunctions of this problem.

We investigate the dependence of the zeros of the eigenfunctions y_n on α and β. Let $0 \le x_n^0 < x_n^1 < ... < x_n^m \le \pi$ be the zeros of the eigenfunction y_n.

We are interested in the following questions:

1) if α is fixed, how do the zeros $x_n^k = x_n^k(\beta)$, $k = 0,1,\ldots,m$ "move",

when β is changed in $[0, \pi)$?
2) if β is fixed, how do the zeros $x_n^k = x_n^k(\alpha)$, $k = 0, 1, \ldots, m$ "move", when α is changed in $(0, \pi]$?
3) how many zeros does the n-th eigenfunction have, i.e. what is m equal?

We answer the questions 1)–3) and, as a corollary, we get a new proof of the Sturm oscillation theorem (the n-th eigenfunction has n zeros in $(0, \pi)$) for arbitrary $q \in L_R^1(0, \pi)$.

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On the Dirichlet boundary value problem in the half-space in weighted spaces

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Let $L^1(\rho)$ be the space of measurable functions in R^2 which are integrable with weight $\rho(x, y) = (1 + |x| + |y|)^\alpha$, $\alpha \geq 0$. The norm in this space is defined as follows:

$$\|f\|_{L^1(\rho)} = \iint\limits_{R^2} |f(x, y)|\rho(x, y)dxdy.$$

We consider the Dirichlet problem in the following formulation: let B be the class of functions U defined in the half-space $\Pi^+ = \{(x, y, z) : z > 0\}$ and satisfying

$$|U(x, y, z)| < M, \quad z > z_0,$$

where $z_0 > 0$ is an arbitrary number, M is a constant dependent on z_0. We want to find a harmonic function $U \in B$ in the half-space Π^+ satisfying

$$\lim_{z \to 0} \|U(x, y, z) - f(x, y)\|_{L^1(\rho)} = 0,$$

where $f \in L^1(\rho)$.

We prove that the problem is solvable if and only if

$$\iint\limits_{R^2} f(u, v)u^k v^m dudv = 0, \quad 0 \leq k + m \leq n - 1,$$

where $n = [\alpha]$.

For arbitrary α the problem in a similar formulation was studied in [1], [2] for the half-plane and disc.

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On the Riemann Boundary Value Problem in the space of continuous functions

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It is well known that Cauchy type integral is a bounded operator in $L^p(1 < p < \infty)$ and Holder functional spaces. This is the most important fact in Riemann boundary value problem research, therefore investigation of this problem in spaces where Cauchy type integral is not bounded operator is significantly harder. For example, when the boundary functions are from L^1 class, the boundary conditions must be posed in the following way [1]: to find a function Φ, $\Phi(\infty) = 0$, analytic in $D^+ \bigcup D^-$ such that the equality

$$\lim_{r \to 1-0} \|\Phi^+(rt) - a(t)\Phi^-(r^{-1}t) - f(t)\|_1 = 0 \tag{1}$$

holds, where $a(t)$ is a piecewise Holder continuous function on the unit circle. It was shown in [1], that this problem is normally solvable and Noetherian in L^1. In this talk we consider the Riemann boundary value problem in the space of continuous functions where Cauchy type integral is not bounded as well. In this case the Riemann problem is formulated in the following way: to determine a function Φ, $\Phi(\infty) = 0$, analytic in $D^+ \bigcup D^-$ such that the equality

$$\lim_{r \to 1-0} \|\Phi^+(rt) - a(t)\Phi^-(r^{-1}t) - f(t)\|_{C(T)} = 0 \tag{2}$$

holds. The necessary and sufficient conditions for normal solvability of the problem were found. In the similar formulation the Riemann problem is considered in arbitrary domains.

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On partial indices of triangular matrix functions

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Let G be an $n \times n$ matrix function defined on a closed curve Γ. Factorization of G is by definition its representation as a product $G_{-}\Lambda G_{+}$ in which G_{+} and G_{-} are boundary values on Γ of matrix functions analytic and invertible in the interior D_{+}(resp., exterior D_{-}) of Γ, subject to some additional conditions on their growth when approaching the boundary. In its turn, $\Lambda(t)$ is diagonal, with the diagonal entries of the form $(t - z_0)^{\kappa_j}$, where z_0 is an (arbitrarily fixed) point of D_{+}. The integers κ_j, called the *partial indices* of G, are defined up to a permutation.

The factorization in general, and the values of the partial indices in particular, play an important role in a wide variety of applied problems, including those to mathematical physics. In this talk we focus on triangular matrix functions G.

For $\chi = (\chi_1, \dots, \chi_n) \in \mathbb{Z}^n$, denote by $T(\chi)$ the class of all lower triangular G with the entries in $L_\infty(\Gamma)$ and i-th diagonal entry factorable with the index χ_i, $i = 1, \dots n$. It is well known (see e.g. [1]) that then G is also factorable, and the set of its partial indices is majorized by χ in the Hardy-Littlewood-Pólya sense. We give a complete description of the set $A(\chi)$ of all n-tuples admissible as partial indices of $G \in T(\chi)$. Conditions are established, in particular, under which the extreme situations occur, that is (i) all n-tuples majorized by χ, or (ii) only those which are permutations of χ, may occur.

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Spectral stability of higher order semi-elliptic operators

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We consider the eigenvalue problem for the operator

$$Hu = \sum_{\substack{(\alpha,\mu)=1 \\ (\beta,\mu)=1}} (-1)^{|\alpha|} D^{\alpha}(A_{\alpha\beta}(x) D^{\beta} u), x \in \Omega$$

subject to homogeneous Dirichlet boundary conditions, where $\mu = (1/m_1, \cdots, 1/m_N)$ is a multi-index and Ω is a bounded open set in $\mathbb{R}^N$. Let $A_{\alpha\beta}$ be bounded measurable real-valued functions defined on Ω satisfying $A_{\alpha\beta} = A_{\beta\alpha}$ and the uniform semi-ellipticity condition

$$\sum_{\substack{(\alpha,\mu)=1 \\ (\beta,\mu)=1}} A_{\alpha\beta}(x) \xi_{\alpha} \xi_{\beta} \geq \theta |\xi|^2$$

We consider open sets Ω for which the spectrum is discrete and can be represented by means of non-decreasing sequence of positive-definite eigenvalues

$$\lambda_1[\Omega] \leq \lambda_2[\Omega] \leq \cdots \leq \lambda_n[\Omega] \leq \cdots$$

Theorem. *Let Ω_1 be an open set in $\mathbb{R}^N$. Then for each $n \in \mathbb{N}$ there exist $c_n, \epsilon_n > 0$ depending only on Ω_1 such that*

$$|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]| \leq c_n \sqrt{d_M(\Omega_1, \Omega_2)},$$

for all $\Omega_2 \subseteq \Omega_1$ such that $d_M(\Omega_1, \Omega_2) < \epsilon_n$, where $d_M(\Omega_1, \Omega_2)$ is the Lebesgue measure of $\Omega_1 \triangle \Omega_2$, the symmetric difference of Ω_1 and Ω_2.

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Essentially nonlinear functional differential equation with advanced argument

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Consider the generalized Emden-Fowler differential equation

$$u^{(n)}(t) + p(t)|u(\sigma(t))|^{\mu(t)} \operatorname{sign} u(\sigma(t)) = 0, \tag{1}$$

where $p \in L_{\mathrm{loc}}(R_+; R)$, $\sigma \in C(R_+; R_+)$, $\sigma(t) \geq t$ and $\mu \in C(R_+; (0, +\infty))$.

We say that the equation (1) is "almost linear" if the condition

$$\lim_{t \to +\infty} \mu(t) = 1$$

is fulfilled, while if

$$\liminf_{t \to +\infty} \mu(t) \neq 1 \quad \text{or} \quad \limsup_{t \to +\infty} \mu(t) \neq 1,$$

then we say that the equation (1) is an essentially nonlinear differential equation.

For "almost linear" and essentially nonlinear differential equations the sufficient (necessary and sufficient) conditions are established for oscillation of solutions. Some results in this direction are given in [1-4].

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Asymptotic of the minimizers of the two-dimensional Mumford-Shah functional near the crack-tip

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We consider the two dimensional Mumford-Shah functional and discuss the situation at the crack-tip. It is well known that the solution asymptotically equals to $\lambda\Im\sqrt{z}$. Under certain conditions we are able to calculate the higher order terms of the asymptotic. The homogeneity orders of the higher order terms can be explicitly calculated. Further we prove that the curvature at the crack-tip cannot be finite unless it is zero.

The isospectral Sturm-Liouville problems

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Let $L\left(q,\alpha,\beta\right)$ denote the Sturm-Liouville problem

$$-y''+q(x)y=\mu y,\quad x\in(0,\pi),\ \mu\in C, \tag{1}$$

$$y(0)\cos\alpha+y'(0)\sin\alpha=0,\quad \alpha\in(0,\pi], \tag{2}$$

$$y(\pi)\cos\beta+y'(\pi)\sin\beta=0,\quad \beta\in[0,\pi), \tag{3}$$

where $q\in L^1_R[0,\pi]$, i.e. q is a real, summable function on $[0,\pi]$.

The *isospectrality problem* is to describe all the problems of the form (1)–(3) that have the same spectrum.

In the series of the papers [1]–[4] E. Trubowitz and others have solved the isospectrality problem when $q\in L^2_R[0,\pi]$.

In [5] the authors have proposed another method based on Gelfand-Levitan integral equation and transformation operators and have solved the isospectrality problem in case when $q'\in L^2_R[0,\pi]$ and $\sin\alpha\neq 0, \sin\beta\neq 0$.

We solve this problem in case when $q\in L^1_R[0,\pi]$ and for arbitrary $\alpha\in\left(0,\pi\right],\beta\in\left[0,\pi\right)$.

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Variational inequalities for a class of nonlinear pseudoparabolic operators

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Let V be a Banach space and K be a nonempty, closed, convex subset of V. A family of operators $A : D(A) \to V'$ is considered with an everywhere dense linear manifold $D(A) \subset V$. The problem is the following: for given $f \in V'$ find $u \in K \bigcap D(A)$ satisfying the inequality

$$(Au, v-u) \geq (f, v-u) \tag{1}$$

where the following conditions are met

- Operator A may be represented in the form $A = L\Lambda + M$,
- $(-\Lambda) : D(A) \to V$ is a generator for linear semigroup $G(s) : V \to V$,
- $L : V \to V'$ is continuous and bounded,
- $(L\Lambda_h(\varphi - \psi), \varphi - \psi) \leq (L\Lambda_h\varphi - L\Lambda_h\psi, \varphi - \psi), \forall \varphi, \psi \in K$,
- $(L\Lambda_h\varphi, \varphi) \geq 0, \forall \varphi \in K$,
- $M : V \to V'$ is pseudomonotone operator in K, i.e. it is bounded in K and the continuous $u_n \rightharpoonup u$ $(u_n \in K)$ and $\overline{\lim}(Mu_n, u_n - u) \leq 0$ imply $\underline{\lim}(Mu_n, u_n - v) \geq (Mu, u - v), \forall v \in V$,
- If the set K is unbounded, then M is coercive in K, i.e. $\exists v_0 \in K$ s.t. $\frac{(Mv, v-v_0)}{\|v\|} \underset{\|v\| \to \infty}{\to} \infty$.

Theorem 1. ***(existence and uniqueness of weak solution)*** *Let all the statements above are true with $v_0 \in K \bigcap D(A)$ in the condition of coercivity of the operator M and the compatibility condition of Λ and K is also satisfied. Then there exists a solution for the corresponding weak problem. If the operator M is strictly monotone, then the weak solution is unique.*

Theorem 2. ***(regularity of the weak solution)*** *Under the assumptions of Theorem 1, let* $int(K) \neq \emptyset$, $V \subset V'$ *and*

- $f \in D(\Lambda)$,
- $(Mu - Mv, u - v) \geq c \|u - v\|^2, \forall u, v \in V$,
- *The semigroup* $G(s)$ *can be extended to the space* V',
- $G(s)(Mv) = M(G(s)v), G(s)(Lv) = L(G(s)v), \forall v \in V, \forall s \geq 0$,
- $\exists \rho > 0, G(s)v + G^*(s)v - G^*(s)G(s)v + (\rho - 1)v \in \rho K, \forall v \in K, \forall s \geq 0$.

Then the weak solution of the variational inequality exists, is unique and satisfies the variational inequality (1).

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H.Weil orthogonal decomposition of the $\mathbf{L_2}(G)$ space and eigenfunctions of the *curl* operator

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Dedicated to the memory of Nazaret Ervandovich Tovmasyan

H.Weil theorem [1] on the orthogonal decomposition of the $\mathbf{L_2}(G)$ space plays a fundamental role in the theory of generalized solutions [2], in the study of the solvability of Navier-Stokes equations [3] and in another questions.

In this work the spectral problems for curl and Stokes operators in a ball B of radius R are solved explicitly. We have used explicit solutions of the Dirichlet and Neumann spectral problems for the Laplace operator [4].

It is proved that the system of vector eigenfunctions of the curl operator (associated with eigenvalues $\lambda = 0$ and $\pm\lambda_\kappa$) form a complete orthogonal basis in $\mathbf{L}_2(B)$.

The eigenfunctions $(\mathbf{v}_\kappa,\ p_\kappa)$ of the Stokes operator in the ball are represented as a sum of two eigenfunctions of the curl operator associated with the opposite eigenvalues: $\mathbf{v}_\kappa = \mathbf{u}^+_\kappa + \mathbf{u}^-_\kappa$, $\mathbf{v}_\kappa|_S = 0$, $\nu \cdot \mathbf{u}^\pm_\kappa|_S = 0$ and $p_\kappa = \text{const}$.

As an example, the boundary value problem $curl\,\mathbf{u}+\lambda\,\mathbf{u} = f,\ \nu\cdot\mathbf{u}|_S = 0$ is solved explicitly by the Fourier method for all λ [5]. See also [6, 7].

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Variation formulas of solution for functional differential equations taking into account delay function perturbation and initial data optimization problems

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For the delay functional differential equation

$$\dot{x}(t) = f(t, x(t), x(\tau(t))), t \in [t_0, t_1]$$

with the continuous initial condition

$$x(t) = \varphi(t), t \in [\tau(t_0), t_0],$$

linear representations of the main part of a solution increment (variation formulas) with respect to perturbations of initial moment t_0, initial function $\varphi(t)$, delay function $\tau(t)$ and right-hand side f are proved. In this paper, moreover, necessary optimality conditions are obtained for the initial data optimization problems. Here, under initial data we imply the collection of the initial moment and function, delay and control functions.

The variation formula plays the basic role in proving of the necessary optimality conditions [1]. Variation formulas for various classes of functional differential equations are given in [1-3].

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A steady glacier flow as an obstacle problem

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This talk is based on the joint work with J.F. Rodrigues.

We discuss models of steady glacier flows in the Orlicz-Sobolev space framework. Following to the approach of Jouvet and Bueler, steady, shallow ice sheet flow is formulated as an obstacle problem with the unknown as the ice upper surface and the obstacle as the underlying bedrock topography. The obstacle problem is written as a variational inequality. The corresponding PDE is highly nonlinear elliptic equation. We show the existence of a solution using a fixed point argument. In some restricted cases the uniqueness of the solution is shown.

Probability and Statistics

Orientation-dependent chord length distribution function for a convex domain

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A family of identities for planar convex domains was discovered by A. Pleijel (see [1]). R. V. Ambartzumian proved these identities (see [2]) and disintegrated Pleijel identities (see [3]) using combinatorial principles in integral geometry. In the classical Pleijel identities integration is over the measure dg in the space $\mathbb{G}$ of lines which is invariant with respect to the all Euclidean motions (see [2]). Let $\mathbb{D}$ be a convex bounded domain in the plane and $[\mathbb{D}] = \{g \in \mathbb{G} : \ \chi(g) = g \cap \mathbb{D} \neq \emptyset\}$. In the paper [4] generalized Pleijel identity for any locally-finite, bundleless measure in the space $\mathbf{G}$ have been proved. This identity is applied to find the so-called orientation-dependent chord length distribution function $F(\mathbb{D}, u, y)$ for a bounded convex domain:

$$b(D,u)\cdot[1-F(\mathbb{D},u,y)] = \frac{1}{2}\int_{[\mathbb{D}]} \delta(|\chi(g)|-y)\,|\chi(g)|\,|\sin(\phi-u)|\,dg-$$

$$-\frac{1}{2}\int_{[\mathbb{D}]} \delta'(|\chi(g)|-y)\cdot|\chi(g)|^2\,|\sin(\phi-u)|\,\cot\alpha_1\,\cot\alpha_2\,dg,$$

where $b(D,u)$ is the breadth function in direction u (the distance between two parallel support lines in direction u), $\delta(y)$ is the Dirac's δ–function concentrated at y, α_1 and α_2 are the angles between the boundary of $\mathbb{D}$ and the line g at the endpoints of $\chi(g)$ which lie in one half-plane with respect to the inside of $\mathbb{D}$, and ϕ is the direction of the line g.

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A solution of the generalized cosine equation in Hilbert's fourth problem

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Hilbert's fourth problem asks for the geometries, defined axiomatically, in which there exists a notion of length for which the line segments are the shortest connections of their endpoints. It was shown in [2] that it is the same to ask: *determine all complete, continuous and linearly additive metrics in* $\mathbf{R}^n$.
The modern approaches make it clear that the problem is at the basis of integral geometry, inverse problems and Finsler geometry. We denote by $\mathbf{E}$ the space of planes in $\mathbf{R}^3$, $\mathbf{S}^2$ - the unit sphere, $[x]$ - the bundle of planes containing the point $x \in \mathbf{R}^3$. Let μ be a locally finite signed measure in $\mathbf{E}$ possessing density $h(e)$ with respect to the invariant measure. Let h_x (defined on $\mathbf{S}^2$) be the restriction of h onto $[x]$. A.V. Pogorelov showed the following result.

Theorem 1. *If H is a smooth linearly additive Finsler metric in $\mathbf{R}^3$, then there exists a uniquely determined locally finite signed measure μ in the space $\mathbf{E}$, with continuous density function h, such that*

$$H(x,\Omega) = \int_{\mathbf{S}^2} |(\Omega,\xi)|\, h_x(\xi)\, d\xi \quad \text{for} \quad (x,\Omega) \in \mathbf{R}^3 \times \mathbf{S}^2. \tag{1}$$

Here h_x is the restriction of h onto $[x]$, $d\xi$ - the spherical Lebesgue measure on $\mathbf{S}^2$.

The measure μ is also called a Crofton measure for the Finsler metric H. Thus, for smooth linearly additive Finsler metrics, Pogorelov's result establish the existence of a Crofton measure, in general not positive. The equation (1), where H is a given even function and h is the unknown, we will call the *generalized cosine equation.*

In [1] we propose an inversion formula for the solution of this integral equation using integral and stochastic geometry methods.

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On differentiability of a flow for an SDE with discontinuous drift

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Consider an SDE of the form

$$\begin{cases} d\varphi_t(x) = a(\varphi_t(x))dt + dw(t), \\ \varphi_0(x) = x, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^d$, $d \geq 1$, $(w(t))_{t\geq 0}$ is a d-dimensional Wiener process, $a = (a^1, \ldots, a^d)$ is a bounded measurable map from $\mathbb{R}^d$ to $\mathbb{R}^d$.

According to [1] there exists a unique strong solution to the equation (1).

We prove the existence of Sobolev derivative of the solution of (1) w.r.t. the initial data when a is a vector function of bounded variation. We show that the derivative is a solution of an integral equation. Solving the equation we see that the derivative can be represented as a function of an additive continuous functional of the process $(\varphi_t(x))_{t\geq 0}$.

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Inference for stress-strength parameter of Levy distribution based on type-II progressive censoring

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In probability theory the *Levy Distribution*, named after Paul Levy, is a continuous probability distribution for a non-negative random variable. It is one of the few distributions that are stable and that have probability density functions that can be expressed analytically. For details see [5].

The probability density function of the *Levy Distribution* is as follows (see, for example, [1], [3]):

$$f(x;\gamma,\delta) = \sqrt{\frac{\gamma}{2\pi}}(x-\delta)^{-\frac{3}{2}}\exp(-\frac{\gamma}{2(x-\delta)}), \tag{1}$$

where $\gamma \in (0,\infty)$ is the scale parameter, $\delta \in (-\infty,\infty)$ is the location parameter and $x > \delta$.

The *Stress-Strength Parameter* $R = P(X < Y)$ is an important reliability parameter arising in the classical Stress-Strength reliability, where one is interested in assessing the proportion of the times the random strength Y of a component exceeds the random stress X to which the component is subjected. There are several works on the estimation of R based on the complete samples (see, for example, [1], [4]). But there is not much attention based on censored samples. Asgharzadeh et al. [2] propose *Stress-Strength Parameter* in Weibull distribution based on progressive censoring. In addition Saracoglu et al. [6] have been estimated the *Stress-Strength Parameter*, when X and Y are independent exponential random variables based on progressively Type-II censored. Among different censoring schemes, we consider progressive Type-II censoring. Progressively Type-II censored sampling is an important method of obtaining data in lifetime studies.

In this paper we are going to study the *Stress-Strength Parameter* R based on progressive Type-II censored samples, where X-sample and Y-sample have the *Levy Distribution* (1), but with different parameters, and two samples of this distributions are independent.

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The entropy of hidden Markov measure of Blackwell's type

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Consider the dynamical system (X, T, μ), where $X = A^Z$ is the full symbolic compact set with the product topology with $A = \{0, 1, ..., d\}$, the shift is $T : \{x_n\} \to \{x_n^{'}\}, x_n^{'} = x_{n+1}$ and the measure μ is a T-invariant hidden Markov probability measure.

For all words $a_1...a_n$ the measure $\mu(a_1...a_n) = \mu(\{x : x_1 = a_1, ..., x_n = a_n\}) = lm_{a_1}...m_{a_n}r$, where the matrices $\{m_0, ..., m_d\}, d \geq 1$ are nonzero substochastic matrices of order J. Moreover, the matrix $P = m_0 + ... + m_d$ is a stochastic matrix, the row l is a left P-invariant probability row and all entries of the column r are equal to 1.

In symbolic dynamics Markov measures have been thoroughly studied. Its entropy is given by an explicit formula.

It is noticeable that the computation of the entropy of hidden Markov dynamical system is a difficult problem. We obtain an explicit formula for the entropy $h(T, \mu)$ of hidden Markov measure of Blackwell's type satisfying $rank(m_a) = 1, a \neq 0$.

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On the asymptotic behavior of distributions of random sequences with random indices and regularly varying "tails"

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In this paper we study the asymptotic behavior of the distribution of the sum of a random number of random variables in the case where the normalizing factors are represented in the form of regularly varying functions. This result generalize the results of [1] - [3].

Definition 1. *A function $F(x) > 0$, measurable on $(0, +\infty)$, is called regularly varying at infinity with index $\alpha \in (-\infty, +\infty)$, if the function $L(t) = t^{-\alpha}F(t), t > 0$ is slowly varying at infinity (SVFI), i.e. for any fixed $x > 0$,*

$$\lim_{t\to+\infty} \frac{L(xt)}{L(t)} = 1$$

Theorem 1. *Let $\{\xi(n)\}_{n=1}^{\infty}$ and $\{\tau_n\}_{n=1}^{\infty}$ be two sequences of random variables, and let their joint local distribution satisfies the following condition:*

$$P\left(\frac{\xi(n) - an^{\alpha}L_1(n)}{bn^{\beta}L_2(n)} < x, \frac{\tau_n - cn^{\gamma}M_1(t)}{dn^{\delta}M_2(t)} < y\right) \underset{n\to\infty}{\to} F(x, y),$$

where $F(x, y)$ is a nonsingular joint limit distribution, for some constants $\alpha > \beta, b \neq 0$, $\gamma > \delta$ and $d \neq 0$, and some SVFI $L_1(t)$, $L_2(t)$, $M_1(t)$ and $M_2(t)$. Then

$$P\left(\frac{\xi(\tau_n) - gn^{\lambda}\mathcal{L}(n)}{hn^{\mu}\mathcal{M}(n)} < x\right) \underset{n\to\infty}{\to} H(x),$$

where the constants g, h, λ and μ, as well as the functions $\mathcal{L}(n), \mathcal{M}(n)$ and distribution function $H(x)$ are described as follows:

I. if $a = c = 0$, then $g = 0$, $\mu = \beta\delta$, $h = bd^{\beta}$, $\mathcal{M} = [M_2(n)]^{\beta} \cdot L_2(dn^{\delta}M_2(n))$, and $H(x) = P(\bar{\xi}\bar{\tau}^{\beta} < x)$, where $\bar{\xi}$ and $\bar{\tau}$ are random variables with joint distribution function $F(x, y)$;

II. if $a \neq 0$, $c = 0$, then $g = 0$, $\mu = a\delta$, $h = ad^{\alpha}$, $\mathcal{M}(n) = [M_2(n)]^{\alpha} \cdot L_1(dn^{\delta}M_2(n))$ and $H(x) = P(\bar{\tau}^{\alpha} < x)$;

III. if $a = 0, c \neq 0$ or $a \neq 0, c \neq 0$ and $\gamma\beta > \gamma(\alpha - 1) + \delta$ then $\lambda = \gamma\alpha$, $g = ac^{\alpha}$, $\mu = \gamma\beta$, $h = bc^{\beta}$, $\mathcal{L}(n) = M_1^{\alpha}(n)L_1(cn^{\gamma}M_1(n))$, $\mathcal{M}(n) = M_1^{\beta}(n)L_2(cn^{\delta}M_1(n))$, and $H(x) = F(x, \infty)$;

IV. if $a \neq 0$, $c \neq 0$ *and* $\gamma\beta < \gamma(\alpha-1)+\delta$ *then* $\lambda = \gamma\alpha$, $g = ac^{\alpha}$, $\mu = \gamma(\alpha-1)+\delta$, $h = ad\alpha c^{\alpha-1}$, $\mathcal{L}(n) = M_1^{\alpha}(n)L_1(cn^{\gamma}M_1(n))$, $\mathcal{M}(n) = M_1^{\alpha-1}(n)M_2(n)\cdot L_1(cn^{\gamma}M_1(n))$ *and* $H(x) = F(\infty, x)$*;*

V. if $a \neq 0$, $c \neq 0$ *and* $\gamma\beta < \gamma(\alpha-1)+\delta$ *, then*

a) provided that

$$\lim_{n\to\infty} \frac{M_1^{\alpha-1}(n)M_2(n)L_1(cn^{\gamma}M_1(n))}{M^{\beta}(n)L_2(cn^{\gamma}M_1(n))} = p^{-1} \quad (0 < p < \infty),$$

$$\lambda = \gamma\alpha, \quad g = ac^{\alpha}, \quad \mu = \gamma\beta, \quad h = 1,$$

$$\mathcal{L}(n) = M_1^{\alpha}(n)L_1(cn^{\gamma}M_1(n)), \quad \mathcal{M}(n) = M_1^{\alpha-1}(n)M_2(n)L_1(cn^{\gamma}M_1(n))$$

and

$$H(x) = P(ad\alpha c^{\alpha-1}\bar{\tau} + bc^{\beta}p^{-1}\bar{\xi} < x);$$

b) if $p = 0$*, then* $h = ad\alpha c^{\alpha-1}$, $\mathcal{M}(n) = M_1^{\alpha-1}(n)L_1(cn^{\gamma}M_1(n))$, $H(x) = F(\infty, x)$*;*

c) if $p = +\infty$*, then* $h = bc^{\beta}$, $\mathcal{M}(n) = M_1^{\beta}(n)L_2(cn^{\gamma}M_1(n))$ *and* $H(x) = F(x, \infty)$*.*

Remark 1. *Under the conditions of the theorem, for uniform numbering of variables* $\xi(n)$ *and* τ_n *their independence is not required.*

Remark 2. *The functions* $\mathcal{L}(n)$ *and* $\mathcal{M}(n)$ *are SVFI.*

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Oracle inequalities for aggregation of affine estimators

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We consider the problem of combining a (possibly uncountably infinite) set of affine estimators in non-parametric regression model with heteroscedastic Gaussian noise. Focusing on the exponentially weighted aggregate, we prove a PAC-Bayesian type inequality that leads to sharp oracle inequalities in discrete but also in continuous settings. The framework is general enough to cover the combinations of various procedures such as least square regression, kernel ridge regression, shrinking estimators and many other estimators used in the literature on statistical inverse problems. As a consequence, we show that the proposed aggregate provides an adaptive estimator in the exact minimax sense without neither discretizing the range of tuning parameters nor splitting the set of observations. We also illustrate numerically the good performance achieved by the exponentially weighted aggregate.

This is a joint work with J. Salmon.

Asymptotic properties of the maximum likelihood estimators for some generalized Pareto-like frequency distribution

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Astola and Danielian (2007), using Birth-Death Processes, constructed some four-parametric *Generalized Pareto-like Frequency Distribution* arising in Bioinformatics as follows:

$$\begin{cases} p_\alpha(x) = [g(\alpha)]^{-1} \cdot \frac{\theta^x}{(x+b)^\rho} \prod_{m=0}^{x-1} \left(1 + \frac{c-1}{(m+b)^\rho}\right),\ x = 1, 2, ..., \\ \\ p_\alpha(0) = [g(\alpha)]^{-1} = \left[1 + \frac{\theta^y}{(y+b)^\rho} \prod_{m=0}^{y-1} \left(1 + \frac{c-1}{(m+b)^\rho}\right)\right]^{-1}. \end{cases} \tag{1}$$

where $\theta \in (0, 1)$, $c \in (0, \infty)$, $b \in (0, \infty)$, $\rho \in (1, \infty)$.

As we see from (1), the model is determined by four parameters: the role of the parameter θ here is explained by Astola and Danielian (2007, Ch. 4, Th. 4.2); the parameter c is a so-called *non-linear scale* parameter (or *exponential scale* parameter); the parameter b is a *location* parameter and the parameter ρ characterizes the *shape* of the probability function.

Farbod and Gasparian (2013) fitted such distribution with the help of two real data sets. They also proposed some conditions of solution coincidence for the system of likelihood equations with the Maximum Likelihood Estimators (MLE) for the parameters. Moreover, the *Accumulation Method* for approximate computation of the MLE with simulation studies have been given in [3].

In this report, under some conditions (see, for example, Borovkov, 1998), the asymptotic properties (weak consistency, asymptotic normality, asymptotic efficiency, asymptotic unbiasedness) of the MLE for the parameters of the model (1) are proposed.

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Asymptotic normality of the logarithm of partial likelihood process in irregular case

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We will present a result concerning the asymptotic normality of the logarithm of Partial Likelihood Process for the sequence of binary statistical experiments with arbitrary filtrations (without “usual” assumptions), extending the corresponding result obtained by J. Jacod in [1].

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Covariogram of a parallelogram

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Let $D \subset R^2$ be a bounded convex body with inner points in the Euclidean plane. The function $C(D,h) = L_2(D \cap (D-h))$, $h \in R^2$ is called the covariogram of D (where $L_2(\cdot)$ is the 2-dimensional Lebesgue measure in R^2), see [1]. For any fixed direction u in the plane we denote by $b(D,u)$ the breadth of D in direction u (the distance between two parallel support lines in direction u). A random line which is parallel to u and intersects D has an intersection point (denote that point by x) with the line which is parallel to direction $u^\perp$($u^\perp$ is the direction perpendicular to u) and passes through the origin. The intersection point x is uniformly distributed over the interval $[0, b(D,u)]$. We can identify the points of interval $[0, b(D,u)]$ and the lines which intersect D and are parallel to u. Denote the orientation-dependent chord length distribution function of D in direction u by

$$F(D,u,t) = \frac{L_1\{x : L_1(g_x(u) \cap D) \leq t)}{b(D,u)}$$

where $g_x(u)$ is the line which is parallel to u and intersects $[0, b(D,u)]$ at the point x, $L_1(\cdot)$ is the 1-dimensional Lebesgue measure. It is shown in [2] that every planar convex body is determined within all planar convex bodies by its covariogram, up to translations and reflections. Using the formula $F(D,t) = (\partial D)^{-1} \int_0^\pi F(D,u,t) \cdot b(D,u)du$ we can calculate the chord length distribution function for D (∂D is the perimeter of D). The explicit forms of the covariogram and the orientation-dependent chord length distribution function are known only for a disc and a triangle, see [3]. We obtain:

1) The forms of the covariogram and the orientation-dependent chord length distribution function for any parallelogram.
2) The form of the chord length distribution function for a parallelogram.

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Set covariance and small-angle scattering intensity of an isotropic set

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The elastic scattering of neutrons, X-rays and synchrotron scattering by a thin, isotropic *two-phase sample material* is closely connected with the concept of the *set covariance* of an *isotropic set*. Hence, results of stochastic geometry are interrelated with experimental *scattering intensity curves* $I(h)$. Here, the variable h denotes the amount of the scattering vector $|\mathbf{h}| = h = 4\pi/\lambda_0 \cdot \sin(\theta/2)$. The variables λ_0 and θ denote the wavelength of the radiation used and the *scattering angle* respectively. Data points $I_k(h_k)$ represent geometric information in reciprocal space.

The connections between the density distribution of the sample $\rho(\mathbf{r_A})$ (in stochastic geometry called *indicator function* $i(r_A)$ of the set) and $I(h)$ are based on first principles of *wave propagation* and *interference*. The amplitude $A(\mathbf{h})$ of the scattered wave results from a volume integral, where the position vector $\mathbf{r_A}$ “samples” the density of the material in real space,

$$A(\mathbf{h}) \sim \int_{V_{irr.}} \rho(\mathbf{r_A}) \exp[-i\ \mathbf{h} \cdot \mathbf{r_A}] dV.$$

The integration region is the whole irradiated sample volume $V_{irr.}$.

The step from $A(\mathbf{h})$ to $I(\mathbf{h})$ can be handled by use of the convolution theorem (see Gille, 2014, pp. 14–19). An isotropic sample results in an isotropic intensity $I(h)$. The parameters *density difference* and *density fluctuation* are introduced. The *real-space structure functions* $Z(\mathbf{r})$ (*function of occupancy*) and $\gamma(\mathbf{r})$ (*SAS correlation function*) result. For the connection to the set covariance $C(r)$, where $Z(r) \equiv C(r)/c$, the distances r between two random sample points A and B, $r = \overline{AB}$, are of importance. Algebraic manipulations lead to the identity $\gamma(r) \equiv [C(r)/c - c]/(1 - c)$, which interrelates the particle volume fraction c, the sample correlation function $\gamma(r)$ and the set covariance $C(r)$ for an isotropic two-phase particle ensemble, $0 < r < L$ and $0 \leq c < 1$. Normalization properties of $\gamma(r)$ are $\gamma(0) = 1$ and $\gamma(\infty) = 0$.

In summary, denoting a point of observation by P, the amplitude of the elastically scattered wave $A(P)$ in point P is directly connected to the set covariance of the sample. Inserting a fixed *order range* L, it holds

$[A(P)]^2 = I(h)$, i.e.,

$$I(h) = \int_0^L 4\pi r^2 \gamma(r) \frac{\sin(hr)}{hr} dr = \int_0^L 4\pi r^2 \left[\frac{C(r)/c - c}{1-c}\right] \frac{\sin(hr)}{hr} dr.$$

Inversion allows to detect $C(r)$ in terms of an experimental curve $I(h)$. This is of practical relevance (see Gille, 2012). There exist connections to the *chord length distributions* (see Aharonyan and Ohanyan, 2012) of the sample (of the set).

There are different denotations of the variables used in different fields (see Gille, 2014, chapters 4–8). For cooperation between mathematicians and physicists, the following remark is of importance: In most textbooks of stochastic geometry, the variable h denotes the random distance between two points A and B. In physics this variable is always denoted by r. The variable h is "reserved" for the scattering vector. However, $\mathbf{r}$ is not a position vector. Furthermore, $C(r)$ is rarely applied in physics. For reasons of normalization, the function $\gamma(r)$ is used instead.

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The Trace Approximation Problem for Toeplitz Operators and Applications

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Let $f(\lambda)$ be an integrable real symmetric function defined on $\mathbb{R} := (-\infty, \infty)$. Given $T > 0$, the *T-truncated Toeplitz operator* generated by $f(\lambda)$, and denoted by $W_T(f)$, is defined by

$$[W_T(f)u](t) = \int_0^T \hat{f}(t-s)u(s)ds, \quad u(s) \in L^2[0,T],$$

where $\widehat{f}(t) = \int_{\mathbb{R}} e^{i\lambda t} f(\lambda)\, d\lambda$, $t \in \mathbb{R}$, is the Fourier-Plancherel transform of $f(\lambda)$. It is clear that

$$\operatorname{tr}[W_T(f)] = \int_0^T \hat{f}(t-t)dt = T\int_{-\infty}^{+\infty} f(\lambda)d\lambda.$$

We pose the following question: what happens when the operator $W_T(f)$ is replaced by a product of Toeplitz operators, or by the inverse of a Toeplitz operator? Observe that the product of Toeplitz operators and the inverse of a Toeplitz operator in general are not Toeplitz operators.

The idea, which goes back to the classical works by G. Szegö is: to approximate the trace of the product of Toeplitz operators (resp. the trace of the inverse of a Toeplitz operator) by the trace of a Toeplitz operator generated by the product of the generating functions (resp. by the trace of a Toeplitz operator generated by the inverse of the generating function), e.g.,

$$\operatorname{tr}\left[W_T(f)W_T^{-1}(g)\right] \approx \operatorname{tr}[W_T(f/g)] = T\int_{-\infty}^{+\infty} f(\lambda)/g(\lambda)d\lambda.$$

The next question that arise, and is of interest in applications, is: how well are the approximations, or more precisely, what are the rates of convergence to zero of the corresponding approximation errors?

Notice that the above questions are also of interest for Toeplitz matrices. All the above can be embedded into the following *trace approximation problem* for Toeplitz operators and matrices.

Let $\mathcal{H} = \{h_1, h_2, \ldots, h_m\}$ be a collection of integrable real symmetric functions defined on the domain Λ, where $\Lambda = \mathbb{R}$ or $\Lambda = (-\pi.\pi]$, and let $A_T(h_k)$ denote either the T-truncated Toeplitz operator, or the $(T \times T)$-Toeplitz matrix, generated by function h_k. Further, let $\tau := \{\tau_k : \tau_k \in \{-1, 1\},\ k = \overline{1, m}\}$ be a given sequence of ± 1's. Define

$$S_{A,\mathcal{H},\tau}(T) := \frac{1}{T}\operatorname{tr}\left[\prod_{k=1}^{m}\{A_T(h_k\}^{\tau_k}\right],$$

$$M_{\Lambda,\mathcal{H},\tau} := (2\pi)^{m-1}\int_\Lambda \prod_{k=1}^{m}[h_k(\lambda)]^{\tau_k}d\lambda,$$

and set

$$\Delta_{A,\Lambda,\mathcal{H},\tau}(T) := |S_{A,\mathcal{H},\tau}(T) - M_{\Lambda,\mathcal{H},\tau}|.$$

The problem is to approximate $S_{A,\mathcal{H},\tau}(T)$ by $M_{\Lambda,\mathcal{H},\tau}$ and estimate the error rate for $\Delta_{A,\Lambda,\mathcal{H},\tau}(T)$ as $T \to \infty$. More precisely, for a given sequence $\tau = \{\tau_k \in \{-1, 1\},\ k = \overline{1, m}\}$, find conditions on functions $\{h_k(\lambda),\ k =$

$\overline{1,m}\}$ such that:

$$\text{Problem (A)}: \quad \Delta_{A,\Lambda,\mathcal{H},\tau}(T) = o(1) \quad \text{as} \quad T \to \infty, \quad \text{or}$$
$$\text{Problem (B)}: \quad \Delta_{A,\Lambda,\mathcal{H},\tau}(T) = O(T^{-\gamma}), \quad \gamma > 0, \quad \text{as} \quad T \to \infty.$$

In this talk we present some results concerning Problems (A) and (B) for Toeplitz operators, in the special case where $\tau_k = 1$, $k = \overline{1,m}$, and some applications to stationary processes.

On a measure with maximal entropy for a suspension flow over a countable state Markov shift

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The subject of the talk belongs to thermodynamic formalism for a countable alphabet Markov shift [1], [2]. (For applications of this theory to smooth dynamics see, e.g., [3] and [4].)

Let $G = (V, E)$ be a directed graph, $V = \mathbb{N}$, and (X, T) the topological Markov shift induced by G, i.e., T is the shift transformation on $X = X(G)$, the space of all the infinite two-sided paths in G (for $x \in X$ we write $x = (x_i, i \in \mathbb{Z})$). Assuming that the graph G is connected, we consider a function $f : X \to [c, \infty)$, $c > 0$, with summable variations and a suspension (or special) flow (X^f, S_t) determined by (X, T) and f (here $X^f = \{(x, u) \in X \times \mathbb{R}_+ : u < f(x)\}$). A key tool in the proof of the main statement in [4] is theorem 2.1 therein that provides conditions on a T-invariant probability measure μ on X under which the S_t-invariant probability measure μ^f on X^f induced by μ in a canonical way has maximal entropy with respect of the flow (X^f, S_t). We note that if such a measure μ exists, it is unique. Moreover, if $f(x)$ depends on x_0 only, then μ can be obtained in an explicit form. Using this fact, we essentially relax the above-mentioned conditions on μ in the general case.

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Linear and planar sections of convex domains

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Let $D \subset R^2$ be a bounded convex body with inner points in the Euclidean plane. The function $C(D,h) = L_2(D \cap (D-h))$, $h \in R^2$ is called the covariogram of D (where $L_2(\cdot)$ is the 2-dimensional Lebesgue measure in R^2), see [1]. For any fixed direction u in the plane we denote by $b(D,u)$ the breadth of D in direction u (the distance between two parallel support lines in direction u). A random line which is parallel to u and intersects D has an intersection point (denote that point by x) with the line which is parallel to direction $u^\perp$($u^\perp$ is the direction perpendicular to u) and passes through the origin. The intersection point x is uniformly distributed over the interval $[0, b(D,u)]$. We can identify the points of interval $[0, b(D,u)]$ and the lines which intersect D and are parallel to u. Denote the orientation-dependent chord length distribution function of D in direction u by $F(D,u,t) = L_1\{x : L_1(g_x(u) \cap D) \le t\}(b(D,u))^{-1}$, where $g_x(u)$ is the line which is parallel to u and intersects $[0, b(D,u)]$ at the point x, $L_1(\cdot)$ is the 1-dimensional Lebesgue measure. The explicit forms of the covariogram and the orientation-dependent chord length distribution function are known only for a disc and a triangle (see [3]). Denote by $\mathbf{E}$ the space of all planes in $\mathbb{R}^3$. Each $e \in \mathbf{E}$ can be introduced by spatial direction ω and the distance p of the plane e from the origin. A random plane with direction ω and intersecting D has an intersection point (denote that point by p) with the line which is parallel to direction ω and passes through the origin. The intersection point p is uniformly distributed over the interval $[0, b(D,\omega)]$, where $b(D,\omega)$ is the distance between two support planes to $D \subset \mathbb{R}^3$ with direction ω. The distribution function of the orientation dependent cross-section area is defined as $\mathcal{F}(D,\omega,t) = L_1\{p : \; L_2\{e_p(\omega) \cap D\} \le t\}(b(D,\omega))^{-1}$, where $e_p(\omega)$ is the plane with direction ω intersecting $[0, b(D,u)]$ at the point p. Orientation-dependent cross-section area distribution function is known only in the case of a ball. We find: 1) $F(D,u,t)$ for a regular polygon and ellipse; 2) $\mathcal{F}(D,\omega,t)$ for an ellipsoid and cylinder. Particularly, in the case of a regular polygon, by integrating $F(D,u,t)$ for all directions u we obtain the result obtained in [2].

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Martingale method in the theory of random fields and its application in statistical physics

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Martingale method is one of the most useful in the theory of random processes, particularly in problems of convergence of sequences of random variables and in limit theorems for sums of random summands. At the same time this method still does not sufficiently employ in the multidimensional problems and particularly in the theory of random fields.

In [1] the notion of martingale–difference random fields was introduced. These random fields are interesting for several reasons: the CLT for such fields is valid under minimal conditions ([2]) and for some class of martingale–difference random fields the exact asymptotic for all moments of sums of their components is known ([3]). This facts allow to prove the convergence of the moments in the central limit theorem for martingale–difference random fields ([3]) and estimate the convergence rate in the central limit theorem for such fields.

Here we present a new approach for proving limit theorems for random fields, which essentially used the above mentioned properties of martingale–differences. This approach allows to widen the range of validity of limit theorems (the central, the functional and the local limit theorems) for random fields. The idea of this approach is the following: using the appropriated randomization we can correspond to a given random field a new one (associated random field), which is a martingale–difference ([4]). There is a connection formula between the finite dimensional probability distributions of the given and associated random fields. By means of this formula one can study the behavior of a given random field by knowing the behavior of associated martingale-difference random field.

This approach can be successfully applied to Gibbs random fields. Particularly one can prove the asymptotical normality of total spin for Ising model outside the critical point. Moreover, in [5] it was presented an example of martingale–difference random field, associated with Ising model, for which the central limit theorem is valid even at a critical point. We hope that the martingale method will allow to investigate the behavior of the total spin at the critical point for the Ising model.

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Compound kernel estimates for some Markov processes

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Consider the integro-differential equation

$$\frac{\partial}{\partial t}u(t,x) = L(x,D)u(t,x), \quad t>0, \quad x\in\mathbb{R}^n, \tag{1}$$

where the operator $L(x,D)$ is defined on functions ϕ from the Schwartz space as

$$L(x,D)\phi(x) := a(x)\nabla\phi(x) + \int_{\mathbb{R}}^{n} \big(\phi(x+u) - \phi(x) - u\nabla\phi(x)1\!\!1_{\{\|u\|\le 1\}}\big)\mu(x,du),$$

and the kernel $\mu(x,du)$ satisfies $\sup_x \int_{\mathbb{R}}^{n}(1\wedge\|u\|^2)\mu(x,du) < \infty$. By developing a version of the parametrix method, we prove the existence of the fundamental solution to (1), and show that this solution gives rise to a Feller process. As an application of our approach, we prove the criterion on the existence of the continuous additive functional corresponding to the process.

The talk is based on the joint work with Aleksei Kulik.

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Classification of linear cocycles: barycenter method

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In this talk we consider a geometric approach to the problem of classification (up to cohomology) of linear cocycles over dynamical systems. Let T be an ergodic measure-preserving automorphism of a probability space $(\Omega,\mathcal{F},\mathsf{P})$. Any measurable function $A\colon\Omega\to GL(l,\mathbb{R})$ generates *a linear cocycle (over T)* which is a random sequence defined by

$$A_n(\omega) := \begin{cases} A(T^{n-1}\omega)\dots A(T\omega)A(\omega), & n\ge 1,\\ \mathrm{Id}, & n=0,\\ A^{-1}(T^{n}\omega)\dots A^{-1}(T^{-1}\omega), & n\le -1. \end{cases}$$

Two such cocycles $A_n(\omega)$ and $B_n(\omega)$ are called *cohomologous* if there exists a measurable function $C\colon\Omega\to GL(l,\mathbb{R})$ such that

$$B(\omega) = C^{-1}(T\omega)A(\omega)C(\omega) \quad \text{a.e.}$$

The paper [1] proves that any $GL(l,\mathbb{R})$-valued cocycle is cohomologous to a block-triangular cocycle with irreducible block-conformal subcocycles on the diagonal (Jordan normal form). A similar result was obtained in [2, 3] in the case $l=2$ with the help of the barycenter method. This method allows one to express the conjugating matrix $C(\omega)$ in explicit form, as opposed to the method in [1] which is based on using Furstenberg's lemma. In this talk we present the extension of the barycenter method to the case of arbitrary l. We give a new construction of Zimmer's linear cover of the supports of so-called ergodic invariant measures on $\mathbb{R}P^{k-1}$ of irreducible $GL(k,\mathbb{R})$-valued cocycles. Using it we reduce an arbitrary linear cocycle to the Jordan normal form with the conjugating matrix expressed via barycenters of measures on the boundary of the symmetric spaces $PGL(m,\mathbb{R})/PO(m)$.

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Iterative searching for the Poisson inter-arrival times of precipitations in Poznań area

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Because of the complexity of meteorological phenomena and their diversity linked to the geographical location there are still worked out the methods to describe (as well as to determinate parameters of) theoretical distributions, which are good enough to model empirical distributions. Such investigations concern the whole Earth as well as concrete regions, and with respect to the rainfall events they are reported, for instance, in papers authored by P.Droogers and J.Hunink (2012), by A.Gevorgyan (2012) and by Astsatryan et al. (2013) relating to Armenia, and in papers by Z.Komar (2010), by M.Czarnecka and J.Nidzgorska-Lencewicz (2011) and by R.Twardosz et al. (2012) dealing with Polish realities.

In this paper we base on data recorded in recent years in the warm half-year (months April-October, when circa 70% of annual precipitations is realized) in 3 measuring stations in Poznań (central Poland) and we look for a statistical description, as a Poisson process, of the distribution of the inter-arrival times of rainfalls. To do it we have to recognize independent rainfall events. It is still not worked out the clear definition what does this notion stands for, the simplest approach is to fix an arbitrarily established time (e.g., two consecutive rainfalls are independent if they are separated by at least one rainless hour).

We take into consideration only the rain itself (which does not mean that we can not formulate proposals relating to specific situations, such as the urban drainage network). First, we discretize data (in a single season and from one measurement station we get a collection of more than quarter of a million records, gathered minute by minute by rain-gauges working continuously). Next, we eliminate isolated insignificant precipitations (i.e., that of the size ≤ 0.2 mm and far enough from closest rains) and we start an iterative procedure (with respect to the first dry time, sequentially increased by 30 minutes) to find the shortest dry period which can be accepted as that separating the independent rainfalls. Having a rainless period equal to m minutes (at the beginning m=45) we count how many times it started raining in every next 30-minute interval, and this way we get the empirical distribution. It produces the estimator to the parameter of the exponential distribution, and we apply Pearson chi-squared test (with the significance level 0.01) on the consistency with the theoretical distribution. If it reveals there is no reason to reject the null hypothesis, we accept m as the minimal gap which separates rainfalls considered as independent ones; otherwise we increase m to have $m + 30$, at the same time we reduce the number of 30-minute intervals by 1, and we repeat the procedure. This approach was advised by Wolfgang Schilling in 1984 (and, as far as we know, it was not applied in the way we do) and in our research it produces m=675 minutes for every examined station. This big value may be surprising (it is rather claimed the gap between the rains is no higher than 2 hours), nevertheless it is contained within the range, which - by using other methods - has been received, in 1994, by Jennifer Wynn.

The reconstruction of specification by its one-point subsystem and justification for Gibbs formula

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The notion of specification (consistent system of probability distributions in finite volumes indexed by infinite boundary conditions) is the basic one in the theory of random fields and in mathematical statistical physics. The theory of description of random fields by means of specification was constructed by Dobrushin ([1, 5, 3]). In his works Dobrushin noted the importance of the problem of finding consistency conditions for a given system of one-point distributions with infinite boundary conditions to be a subsystem of some specification. The answer to this problem would not only permit one to reformulate Dobrushin's theory in terms of one-point conditional distributions, but also develop the theory in various directions.

Here we propose a solution to that problem by giving necessary and sufficient consistency conditions for a given system of one-point distributions to be a subsystem of some specification ([4, 5]). Also we give probabilistic, algebraic and physical interpretations of these conditions ([6]). We show that considering physical interpretation gives the possibility to justify the known Gibbs formula (distribution) by the variation principle.

The description of specifications and random fields by means of one-point conditional distributions gave a possibility to formulate a Gibbsianness criteria in terms of one-point conditional distribution ([7]). These criteria allows one to develop an alternative approach to the Gibbs theory by giving a probabilistically explicit definition of Gibbs random field. Here we present some introductory steps in this direction.

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Tomography problems of pattern recognition

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The present report contains a review of the main results of Yerevan research group in tomography of planar bounded convex domains. Let $\mathbb{D}$ be a convex body in n-dimensional Euclidean space R^n and $L_n(\cdot)$ be the n-dimensional Lebesgue measure. The covariogram of a $\mathbb{D}$ is defined for $h \in R^n$ by $C(D,h) = L_n(D \cap (D-h))$. G. Matheron conjectured that a planar convex body is uniquely determined by its covariogram. In the plane, an affirmative answer to the covariogram problem for convex bodies was given in [1]. Very little is known regarding the covariogram problem when the space dimension is larger than 2. It is known that centrally symmetric convex bodies in any dimension are determined by their covariogram, up to translations. Bianchi (see [1]) found counterexamples to the covariogram conjecture in dimensions greater than or equal to 4, and a positive answer for three-dimensional polytopes. The general three-dimensional case is still open. Denote by $F(\mathbb{D}, u, x)$ the orientation-dependent chord length distribution function. Obtaining the explicit form of the covariogram for any convex body is very difficult problem, but we can obtain covariograms for a subclass of convex bodies. These forms help us to solve many probabilistic problems, in particular calculate chord length distribution functions $F(\mathbb{D}, x)$ and $F(\mathbb{D}, u, x)$ (see [2]). It is proved in [2] that for any finite subset A from S^1 (S^1 is the circle of unit radius centered at the origin), there are two non-congruent domains for which orientation-dependent chord length distribution functions coincide for any direction from A. Moreover, in [2] explicit forms for covariogram and $F(\mathbb{D}, u, x)$ for arbitrary triangle are obtained. Finally, if we have the values of $F(\mathbb{D}, u, x)$ for a dense set from S^1, then we can uniquely recognize a triangle (see [2]).

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On the limit behavior of a sequence of Markov processes with irregular behavior at a fixed point

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We study the limit behavior of a sequence of Markov processes $\{X_n\}$ such that their distributions outside any neighborhood of a fixed "singular" point attracts to some law. In a neighborhood of this point a behavior of $\{X_n\}$ may be irregular.

As an example of the general results, we consider a limit of a sequence $\{X_n(t) = \frac{S([nt])}{\sqrt{n}},\ n \geq 1\}$, where $\{S(n),\ n \geq 0\}$ is a Markov chain on $\mathbb{Z}$ with transition probabilities:

$$p_{i,i\pm 1} = 1/2,\ |i| > m,$$

and with finite expectation of a jump for $|i| \leq m$:

$$\sum_j p_{ij}|j| < \infty,\ |i| \leq m.$$

The limit of this sequence is a skew Brownian motion [1], i.e. a continuous Markov process with transition density

$$p_t(x, y) = \varphi_t(x - y) + \gamma \mathrm{sign}(y)\varphi_t(|x| + |y|),\ x, y \in \mathbb{R},$$

where $\gamma \in [-1, 1]$, $\varphi_t(x) = \frac{1}{\sqrt{2\pi t}} e^{-x^2/2t}$ is a density of the normal distribution $N(0, t)$. A parameter γ is calculated explicitly in terms of transition probabilities $\|p_{ij}\|$. Note that in contrast to [2, 3, 4, 5] we do not assume boundedness of jumps if $|i| \leq m$.

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Cluster expansions and construction of point processes in statistical mechanics

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Using a new approach which is based on the method of cluster expansions we reconsider the problem of construction of interacting point processes. For a large class of signed locally finite measures L defined on the space of finite configurations in the underlying phase space X, we construct the associated point process P_L on X i.e. a probability measure on the space of locally finite configurations of X.

In special case, where L is given in terms of Ursell functions defined for some underlying pair potential, we obtain point processes of classical statistical mechanics, also processes which are associated to continuous quantum systems in Feynman - Kac representation.

Moreover, for the case of non negative pair potential we prove that these processes are the Gibbs processes in the sense of Dobrushin, Lanford and Ruelle.

Under natural conditions on the pair interaction we prove also the existence of the Gibbs modifications of the processes P_L.

(A joint work with Benjamin Nehring and Hans Zessin)

Limit theorems for mixed stochastic differential equations

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The main object of the talk will be so-called mixed stochastic differential equation in $\mathbb{R}^d$:

$$X_t = X_0 + \int_0^t a(X_s)ds + \int_0^t b(X_s)dW_s + \int_0^t c(X_s)dB_s^H, \qquad (1)$$

where $W = \{W_t, t \geq 0\}$ is a standard Wiener process, $B = \left\{B_t^H, t \geq 0\right\}$ is a fractional Brownian motion with the Hurst parameter $H > 1/2$.

The motivation to consider such equations comes from financial mathematics, where it is useful to distinguish between two main sources of randomness. The first source is the stock exchange itself with thousands of agents; the noise coming from this source can be assumed white and is best modeled by a Wiener process. The second source is the financial and economical background. The random noise coming from this source usually has a long range dependence property, which can be modeled by a fractional Brownian motion B^H with the Hurst parameter $H > 1/2$.

In my talk I will concentrate on limit theorems for equations (1), including stability of solutions under convergence of noises and coefficients and convergence of Euler approximations.

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The probabilistic approximation of the one-dimensional initial-boundary value problem solution

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We consider the equation

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2}\frac{\partial^2 u}{\partial x^2} + f(x)u,$$

where σ is a complex-valued parameter satisfying $\mathrm{Re}\sigma^2 \geq 0$. When σ is real, this equation corresponds to the Heat equation, while for $\mathrm{Re}\sigma^2 = 0$ it corresponds to the Schrödinger equation. For this equation we consider the initial-boundary value problem with Dirichlet conditions

$$u(0,x) = \varphi(x),\ u(t,a) = 0,\ u(t,b) = 0.$$

In the case when σ is real, there exists a probabilistic representation of the solution in the form of the mathematical expectation (the so called Feynman-Kac formula), namely,

$$u(t,x) = \mathrm{E}\left\{\varphi(\widetilde{\xi}_x(t\wedge\tau))e^{\int_0^{t\wedge\tau} f(\widetilde{\xi}_x(v))dv}\right\},$$

where $\widetilde{\xi}_x(t)$ is a Brownian motion with a parameter σ, killed at the exit time τ from the interval $[a,b]$. Basing on this representation, one can approximate the solution using some suitable approximation of the Wiener process. This approach doesn't work if $\mathrm{Im}\sigma \neq 0$. It is known that when σ is not a real number, no analogue of the Wiener measure exists, and hence one can not present the Feynman -Kac formula as an integral with respect to a σ-additive measure in a trajectory space. When $\mathrm{Re}\sigma^2 = 0$ (that corresponds to the Schrödinger equation) one can use an integral with respect to the so called Feynman measure, which is a finitely-additive complex measure in the trajectory space defined as a limit over a sequence of partitions of an interval $[0,T]$. It should be mentioned that this approach is not a probabilistic one in the usual sense, since the very notion of the probability space does not appear in it. To get the stochastic approximation of the solution we use another approach based on the theory of generalized functions. On a special probability space we define the sequence of probability measures $\{\mathrm{P}_n\}$ and the limit object $\mathrm{L} = \lim_{n\to\infty} \mathrm{P}_n$, which is not a measure but only a generalized function. This means that the convergence $\int f d\mathrm{P}_n \to (\mathrm{L}, f)$ is valid only for test functions f. On this probability space we define a complex-valued process so that the mathematical expectation with respect to the measure P_n of some functional of this process converges to the value of the generalized function applied to this test function, leading to the solution of the initial-boundary value problem.

On random sections of octahedra

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In Euclidean space $\mathbb{R}^3$ we consider a family of convex octahedra. Octahedron is a polyhedron with 8 triangle faces, 12 edges and 6 vertices. Each vertex is the common point of four edges.

The sections of convex octahedra by random planes are studied. The distribution of random planes is generated by the measure of the planes, which is invariant with respect to Euclidean motions in $\mathbb{R}^3$. The probabilities P_n of events that these sections are n-gon are considered, $n = 4, 6$.

Using R.Ambartzumian's combinatorial formula (see [1]) the probabilities P_n are calculated. It is proved that P_4 reaches his minimal value and P_6 reaches his maximal value for regular octahedron. The asymptotic behavior of these probabilities is investigated as the altitude trends to zero or infinity. For these probabilities we obtain the inequalities, which hold for any altitude. The similar results for parallelepipeds was obtained in [2].

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On isotopic semirings

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Definition 1. *A semiring is a set R equipped with two binary operations $+$ and $\cdot$ called addition and multiplication, such that*
1. $(R, +)$ is a commutative monoid with identity element 0:

1. $(a + b) + c = a + (b + c)$
2. $0 + a = a + 0 = a$
3. $a + b = b + a$

2. $(R, \cdot)$ is a monoid with identity element 1:

1. $(ab)c = a(bc)$
2. $1a = a1 = a$

3. Multiplication left and right distributes over addition:

1. $a(b + c) = (ab) + (ac)$
2. $(a + b)c = (ac) + (bc)$

4. Multiplication by 0 annihilates R:

1. $0a = a0 = 0$

Definition 2. *A semiring $(R, +, \cdot)$ is called commutative, if $(R, \cdot)$ is a commutative groupoid.*

For two concepts of isotopic algebras considered in [1] and [2] we prove the following result.

Theorem 1. *Isotopic semirings are isomorphic.*

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Relative derived categories and their equivalences

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In this talk we study relative derived category of an abelian category $\mathcal{A}$ with respect to a contravariantly finite subcategory $\mathcal{C}$. We mainly focus on the case where $\mathcal{A}$ has enough projectives and $\mathcal{C}$ is the class of Gorenstein projective objects. In this case the $\mathcal{C}$-relative derived category is called Gorenstein derived category.

Completeness of automorphisms groups of free periodic groups

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A group is called *complete*, if it has a trivial center and each of its automorphisms is inner. If the center of a group G is trivial, then it is embedded into the group of its automorphisms $Aut(G)$. This allows to consider the automorphism tower

$$G = G_0 \lhd G_1 \lhd \cdots \lhd G_k \lhd \cdots,$$

where $G_k = Aut(G_{k-1})$ and G_k is identified with $Inn(G_k)$ under the embedding $G_k \hookrightarrow Aut(G_k)$, $g \mapsto i_g$ $(k = 1, 2, ...)$. It is clear that if the group of automorphisms of centerless group G_0 is complete, then its automorphism tower terminates after the first step. According to classical Wielandt's theorem, the automorphism tower of any finite centerless group terminates after a finite number of steps. For infinite groups the analogous statement is false. In 1975 J. Dyer and E. Formanek in [1], [2] proved that if F is a free group of finite rank > 1, then its group of automorphisms $Aut(F)$ is complete. V. Tolstikh in [3] proved the completeness of $Aut(F)$ for free groups F of infinite rank.

We will establish that the automorphism tower of non-cyclic free Burnside group $B(m, n)$ is terminated on the first step for any odd $n \geq 1003$. Hence, the automorphism tower problem for groups $B(m, n)$ is solved. We show that it is as short as the automorphism tower of the absolutely free groups. In particular, the group $Aut(B(m, n))$ is complete.

Theorem 1. *The automorphisms group $Aut(B(m, n))$ of the free Burnside group $B(m, n)$ is complete for any odd $n \geq 1003$ and $m > 1$.*

Theorem 2. *The automorphisms groups $Aut(B(m, n))$ and $Aut(B(k, n))$ are isomorphic if and only if $m = k$ (for any odd $n \geq 1003$).*

Theorem 3. *For any odd $n \geq 1003$ and $m > 1$, the group of all inner automorphisms $Inn(B(m,n))$ is the unique normal subgroup of the group $Aut(B(m,n))$ among all subgroups, which are isomorphic to a free Burnside group $B(s,n)$ of some rank s.*

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On some rational and integral complex genera

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The general four variable complex elliptic genus ϕ_{KH} on $MU_* \otimes \mathbb{Q}$ is defined in [5],[6] to satisfy the following property: if one denotes by $f_{Kr}(x)$ the exponent of the Krichever [2], [3], [6] universal formal group law $\mathcal{F}_{Kr}$, then the series $h(x) := f'_{Kr}(x)/f_{Kr}(x)$ satisfies the differential equation $(h'(x))^2 = S(h(x))$, where $S(x) = x^4 + p_1x^3 + p_2x^2 + p_3x + p_4$, the generic monic polynomial of degree 4 with formal parameters p_i of weights $|p_i| = 2i$.

To generalize naturally Oshanin's elliptic genus from $\Omega_*^{SO} \otimes \mathbb{Q}$ to $\mathbb{Q}[\mu, \epsilon]$ [8], new elliptic genus ψ is defined in [9] to be the genus $\psi : MU_* \otimes \mathbb{Q} \to \mathbb{Q}[p_1, p_2, p_3, p_4]$ whose logarithm is equal to

$$\int_0^x \frac{dt}{\omega(t)}, \quad \omega(t) = \sqrt{1 + p_1t + p_2t^2 + p_3t^3 + p_4t^4},$$

and p_i are again formal parameters, $|p_i| = 2i$.

Clearly, to calculate the values of ψ on CP_i, the generators of the rational complex bordism ring [7] $MU_* \otimes \mathbb{Q} = \mathbb{Q}[CP_1, CP_2, \dots]$ we need only the Taylor expansion of $(1+y)^{-1/2}$ as by above definition $log'_\psi = \sum_{i\geq 1} \psi(CP_i)x^i$.

It is natural to ask if there exists similar elementary method, different from formulas in [4], [5], for calculation of ϕ_{KH}. Our answer is the formula

$$\phi_{KH} = \psi \circ \kappa^{-1},$$

where ψ is the genus mentioned above and κ classifies the formal group law strictly isomorphic [7] to the universal formal group law under strict isomorphism $\nu(x) = xCP(x)$. The values $\kappa(CP_i)$ are determined by equating the coefficients at x^i in $\sum_{i\geq 1} \frac{\kappa(CP_i)}{i+1} x^{i+1} = \sum_{i\geq 1} \frac{CP_i}{i+1} (\nu^{-1}(x))^{i+1}$.

We construct certain elements A_{ij} in the Lazard ring and give an alternative definition of the universal Buchstaber [2] formal group law. This implies that the universal Buchstaber and Krichever formal group laws coincide and the corresponding coefficient ring is the quotient of the Lazard ring by the ideal generated by all A_{ij}, $i, j \geq 3$.

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On hypergroups over the group and extensions of a group

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A hypergroup over the group (for the original definition see [1]) is an algebraic object, consisting of the database (B0) - (B4), satisfying the axioms (A1) - (A5). The database is the following.

(B0) A (multiplicative) group H.

(B1) A right quasigroup M with the binary operation

$$\Xi : M \times M \to M, \quad ab \to [a, b]$$

and with a (unique!) left neutral element.

(B2) A "scalar product on M with values in H", that is a mapping

$$\Lambda : M \times M \to H, \quad ab \to (a, b).$$

(B3) A right action of the group H on the set M:

$$\Phi : M \times H \to M, \quad a\alpha \to a^{\alpha}.$$

(B4) A "left action of M on H", that is a mapping

$$\Psi : M \times H \to H, \quad a\alpha \to {}^{a}\alpha.$$

We demand that this mapping was surjective.

The list of the axioms is the following.

(A1) ${}^{a}(\alpha \cdot \beta) = {}^{a}\alpha \cdot {}^{a^{\alpha}}\beta$;
(A2) $[a, b]^{\alpha} = [a^{{}^{b}\alpha}, b^{\alpha}]$;
(A3) $(a, b) \cdot {}^{[a,b]}\alpha = {}^{a}({}^{b}\alpha) \cdot (a^{{}^{b}\alpha}, b^{\alpha})$;
(A4) $[[a, b], c]) = [a^{(b,c)}, [b, c]]$;
(A5) $(a, b) \cdot ([a, b], c) = {}^{a}(b, c) \cdot (a^{(b,c)}, [b, c])$.

This hypergroup is denoted by ${}_{H}M$. An action $\mathcal{A}$ of the group

$$H^{M} = \{\lambda : M \to H\}$$

is defined on the set $Hg(H, m)$ of classes of isomorphic hypergroups ${}_{h}M$ with cardinality $|M| = m$.

Theorem 1. *Let $Hg(H, m)/H^{M}$ be the set of orbits with respect of action $\mathcal{A}$ and $Ext(H, m)$ be the set of classes of isomorphic extensions G of the group H of index m. Then there exists a canonical isomorphism between $Hg(H, m)/H^{M}$ and $Ext(H, m)$.*

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On regular medial division n-ary groupoids

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The algebra (Q, Σ) is said to be *medial*, if it satisfies the mediality hyperidentity, i.e. for any $f, g \in \Sigma$:

$$f(g(x_{11},\ldots,x_{1n}),\ldots,g(x_{m1},\ldots,x_{mn}))=g(f(x_{11},\ldots,x_{m1}),\ldots,f(x_{1n},\ldots,x_{mn})).$$

In particular, the n–ary groupoid $Q(f)$ is said to be *medial*, if it satisfies the identity

$$f(f(x_{11},...,x_{1n}),...,f(x_{n1},...,x_{nn}))=f(f(x_{11},...,x_{n1}),...,f(x_{1n},...,x_{nn})).$$

Let $Q()$ be an n–groupoid. Denote by $\overline{a}$ the sequence $a_1^n \in Q^n$. Denote by $L_i(\overline{a})$ the map of Q to Q such that

$$L_i(\overline{a})x = (a_1...a_{i-1}xa_{i+1}...a_n) = (a_1^{i-1}xa_{i+1}^n),$$

for all $x \in Q$. The map $L_i(\overline{a})$ is called an *i–translation* with respect to a.

The n–groupoid $Q()$ is called a division (quasigroup) if every $L_i(\overline{a})$ is a surjection (bijection) for all $\overline{a} \in Q^n$ and all $i = 1,...,n$.

The n–groupoid $Q()$ is called *i–regular* if $L_i(\overline{a}) = L_i(\overline{b})$, whenever $\overline{a}, \overline{b} \in Q^n$ and $L_i(\overline{a})c = L_i(\overline{b})c$, for some $c \in Q$. The n– groupoid $Q()$ is called *regular* if it is i–regular, for all $i = 1,...,n$.

Theorem. *Let $Q()$ be a regular medial division n–groupoid. Then there exist an abelian group, $Q(+)$, its pairwise commuting surjective endomorphisms, $\alpha_1,...,\alpha_n$, and a fixed element b of the set, Q, such that*

$$(x_1x_2...x_n) = (x_1^n) = \alpha_1x_1 + \alpha_2x_2 + ... + \alpha_nx_n + b,$$

for all $x_i \in Q$, $i = 1,\ldots,n$.

Theorem. *Finely generated medial division n-ary groupoid is an n-ary quasigroup.*

Periodic groups saturated by direct products of Suzuki groups and elementary Abelian 2-groups

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Let $\mathfrak{M}$ be a non-empty set of finite groups. We say that a group G is *saturated* by groups from $\mathfrak{M}$, iff every finite subgroup of G is contained in a subgroup of G, isomorphic to an element of $\mathfrak{M}$ [3].

One can find a review of the results on the structure of groups saturated by various sets of groups in [1].

In particular, K. Philippov [2] showed, that a periodic group saturated by simple Suzuki groups is isomorphic to a simple Suzuki group over a locally finite field of characteristic 2.

We generalize this result as follows.

Suppose that $\mathfrak{M} = \{Sz(2^{2m+1}) \times V_n \mid m = 1, 2, ..., n = 1, 2, ...\}$, where V_n is an elementary Abelian 2-group of order 2^n.

Theorem. *If G is a periodic group saturated by $\mathfrak{M}$, then $G \simeq P \times V$, where V is an elementary Abelian 2-group, and $P \simeq Sz(Q)$ for some locally finite field Q of characteristic 2. In particular, G is locally finite.*

The work is supported by RFBR (grant 09-01-00717-a).

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On algebras with medial-like hyperidentities

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Let us consider the following hyperidentities:

$$g(f(x,y), f(u,v)) = f(g(x,u), g(y,v)), \quad \text{(Mediality)} \tag{1}$$

$$g(f(x,y), f(u,v)) = f(g(v,y), g(u,x)), \quad \text{(Paramediality)} \tag{2}$$

$$g(f(x,y), f(u,v)) = g(f(x,u), f(y,v)), \quad \text{(Co-mediality)} \tag{3}$$

$$g(f(x,y), f(u,v)) = g(f(v,y), f(u,x)), \quad \text{(Co-paramediality)} \tag{4}$$

$$g(f(x,y), f(y,v)) = f(g(x,u), g(u,v)), \quad \text{(Intermediality)} \tag{5}$$

$$g(f(x,y), f(u,y)) = f(g(x,v), g(u,v)), \tag{6}$$

$$g(f(x,y), f(u,y)) = f(g(x,v), g(v,u)), \tag{7}$$

$$f(x,x) = x. \quad \text{(Idempotency)} \tag{8}$$

The algebra (A, F) is called:

- medial, if it satisfies the hyperidentity (1),
- paramedial, if it satisfies the hyperidentity (2),
- co-medial, if it satisfies the hyperidentity (3),
- co-paramedial, if it satisfies the hyperidentity (4),
- idempotent, if it satisfies the hyperidentity (8),

for every $f, g \in F$. The algebra (A, F) is called mode, if it is medial and idempotent.

The following characterization of medial algebars with quasigroup operations was obtained by Yu. M. Movsisyan (Transitive Modes. *Demonstratio Mathematica.* **XLIV** (2011), No. 3, 511-522.).

Theorem 1. *Let (Q, F) be a binary medial algebra with quasigroup operations, then there exists an abelian group $(Q, +)$ such that every operation $f_i \in F$ determined by the rule:*

$$f_i(x, y) = \alpha_i x + \beta_i y + c_i,$$

where α_i, β_i are automorphisms of the group $(Q, +)$, and $c_i \in Q$ is a fixed element. The group $(Q, +)$ is unique up to isomorphisms. Moreover, if (Q, F) is a mode, then

$$f_i(x, y) = \alpha_i x + \beta_i y,$$

where α_i, β_i are automorphisms of both the group $(Q, +)$ and of the algebra (Q, F).

In this talk we consider the algebras with quasigroup operations such that their operations satisfy the hyperidentites $(2-7)$, and we obtain similar characterization.

Groups with certain conditions on fixed points of automorphisms

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Let G be a group and α be an automorphism of G. We denote the fixed points of α in G by $C_G(\alpha)$. The structure of $C_G(\alpha)$ gives a lot of information about the structure of G.

Thompson proved that a finite group with a fixed point free automorphism of prime order is nilpotent. An automorphism ϕ of G is called a **splitting automorphism** if for every $x \in G$

$$xx^{\phi}x^{\phi^2}...x^{\phi^{n-1}} = 1,$$

where n is the order of ϕ. Clearly, a fixed point free automorphism of a finite group is a splitting automorphism. Moreover, if α is a splitting automorphism of a group G, then $C_G(\alpha)$ has exponent dividing n.

Kegel generalized Thompson's result to splitting automorphisms, namely he proved that a finite group with a splitting automorphism of prime order is nilpotent. Rowley proved that a finite group with a fixed point free automorphism is solvable. However, there are non-solvable finite groups admitting a splitting automorphism.

Example. *Observe that the cyclic group* $\mathbb{Z}_{31}$ *has a fixed-point-free automorphism* α *of order* 30. *Now, define* $G = \mathbb{Z}_{31} \times A_5$ *and consider*

$$\phi: \ \mathbb{Z}_{31} \times A_5 \longrightarrow \mathbb{Z}_{31} \times A_5$$
$$(x, y) \longrightarrow (x^{\alpha}, y).$$

One can observe that ϕ *is a splitting automorphism of* G *of order* 30, *but* G *is not solvable.*

By Kegel's result, a finite group admitting a splitting automorphism of prime order is nilpotent. Moreover, Jabara proved that a finite group with a splitting automorphism of order 4 is solvable. Hence, one might ask the following question:

Question. *Let* m *be a natural number which is not divisible by the exponent of any non-abelian finite simple group. Let* G *be a finite group admitting a splitting automorphism of order* m. *Is* G *necessarily solvable?*

In this talk we will give a partial answer to this question. In particular, we will prove the following result:

Theorem 1. *[1] A finite group with a splitting automorphism of odd order is solvable.*

Moreover, we will give a survey of recent results obtained imposing certain conditions on the structure of fixed points of automorphisms when G is a locally finite group.

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On interrelation between topological and dynamical properties of diffeomorphisms on 3-manifolds

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The report is devoted to exposition of results on dynamics of diffeomorphisms with hyperbolic nonwandering set in dimension 3 obtained in recent time by the author in collaboration with Russian (V. Medvedev, O. Pochinka, E. Zhuzoma) and French (C. Bonatti, F Laudenbach) mathematicians.

We will discuss the following questions:

topological classification of diffeomorphisms with finite and infinite hyperbolic nonwandering set;

interrelation between dynamics and topology of ambient manifold;

existence of global Lyapunov function whose set of critical points coincide with nonwandering set (such function is called energy function);

existence of simple arc joining two Morse-Smale diffeomorphisms on 3-manifold.

The surveys of the results and references can be found in [1], [2].

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On Morse-Smale cascades that embed in topological flows

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Results under presentation are obtained in collaboration with V. Grines and O. Pochinka, and published in [1].

A C^r-diffeomorphism (cascade) $f : M^n \to M^n$, $r \geq 1$, on smooth connected closed manifold of dimension n embeds in a C^l-flow, $l \leq k$, if f is the time-one map of such a flow. Palis showed in [2] that the set of C^r-diffeomorphisms that embed in C^1-flows is a set of first category. At the same time, the structural stability of Morse-Smale diffeomorphisms leads to the existence of open sets of cascades that embed in topological flow (for example, a neighbourhood of time-one map of a Morse-Smale flow). In [3] the following necessary conditions of embedding of Morse-Smale cascade f in a topological flow were stated:

(1) non-wandering set of f consists only of fixed points;

(2) f restricted to each invariant manifold of its fixed points is orientation preserving;

(3) for any fixed points p, q having non-empty intersection of invariant manifolds, the intersection does not contain compact components.

It was also shown in [3] that conditions (1)-(3) are sufficient for $n = 2$.

In the dimension $n = 3$ an additional obstacle for Morse-Smale cascade to embed in a flow is the non-trivial embedded separatrices of saddle points. To describe them, we put into a correspondence to each diffeomorphism f a global scheme S_f which is a set of wandering orbits with projections of 2-dimensional invariant manifolds. We introduce a definition of triviality of global scheme and prove the following theorem.

Theorem. *A Morse-Smale cascade $f : M^3 \to M^3$ embeds in continuous flow iff its scheme is trivial.*

We also discuss a possibility of expanding our result for Morse-Smale cascades on manifolds of dimension $n > 3$.

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On the Pólya problem of conversion between permanents and determinants

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The talk is based on the works [1, 2, 3].

While the computation of the determinant can be done in a polynomial time, it is still an open question, if there exists a polynomial algorithm to compute the permanent. Due to this reason, starting from the work by Pólya, 1913, different approaches to convert the permanent into the determinant were under the intensive investigation.

Among our results we prove the following theorem:

Theorem 1. *Suppose $n \geq 3$, and let F be a finite field with* $\operatorname{char} F \neq 2$. *Then, no bijective map $T : M_n(F) \to M_n(F)$ satisfies* $\operatorname{per} A = \det T(A)$.

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Gorenstein projective, injective and flat sheaves over posets

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In this talk we study the category of sheaves over an infinite partially ordered set with its natural topological structure. Totally acyclic complexes in this category will be characterized in terms of their stalks. This lead us to describe Gorenstein projective, injective and flat sheaves. As an application, we get an analogue of a formula due to Mitchell, giving an upper bound on the Gorenstein global dimension of such categories. Based on these results, we present situations in which the class of Gorenstein projective sheaves is precovering as well as situations in which the class of Gorenstein injective sheaves is preenveloping.

Algebras with fuzzy operations and fuzzy equalities

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The problem of development of algebras with fuzzy operations is formulated in ([1], P:136). In this paper we introduce algebras with fuzzy operations. We consider fuzzy operations instead of ordinary functions in structure of algebras and investigate properties of such algebras.

We will use complete residuated lattices $\mathcal{L} =< L; \wedge, \vee, \otimes, \to, 0, 1 >$ as the structures of truth values.

Definition. *Let $\approx^{\mathbf{M}}$ be a fuzzy equality on M. An (n+1)-ary fuzzy relation ρ on a set M is called an n-ary fuzzy operation w.r.t. $\approx^{\mathbf{M}}$ and $\approx^{\mathbf{M^n}}$ if we have the following conditions:*
Extensionality:

$$(p \approx^{\mathbf{M^n}} p') \otimes (y \approx^{\mathbf{M}} y') \otimes \rho(p, y) \leq \rho(p', y') \quad \forall p, p' \in M^n,\ \forall y, y' \in M,$$

Functionality:

$$\rho(p,y) \otimes \rho(p,y') \leq y \approx^{\mathbf{M}} y' \quad \forall p \in M^n,\ \forall y, y' \in M,$$

Fully-defined:

$$\bigvee_{y \in M} \rho(p,y) = 1 \ \ \forall p \in M^n,$$

where $(a_1, \cdots, a_n) \approx^{M^n} (b_1, \cdots, b_n) = \bigwedge_{i=1}^{n} (a_i \approx^M b_i)$. *We say that* ρ *is a fuzzy operation on* M *with arity* n.

Definition. *An algebra with fuzzy operations of type* $\langle \approx, F \rangle$ *is a triplet* $\mathcal{M} = \langle M, \approx^{\mathcal{M}}, \mathcal{F}^M \rangle$ *such that* $\approx^{\mathcal{M}}$ *is a fuzzy equality on the set* M *and* $\mathcal{F}^{\mathcal{M}}$ *is the set of fuzzy operations on the set* M.

Formulas of fuzzy equational logic are the set

$$Fml = \{\otimes_{i \in I} t_i \Rightarrow s | t, t' \in T(X), s \in S\},$$

where $S = \{x \approx y | x, y \in X\}$.

A theory is a set T of formulas, i.e. $T \subseteq Fml$.

Theorem. (Compactness Theorem). *A Theory* T *has a model iff each it finite subtheory has a model.*

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On surfaces of constant astigmatism

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Surfaces of constant astigmatism, i.e. surfaces characterized by the condition $\rho_2 - \rho_1 = \text{const} \neq 0$, where ρ_1, ρ_2 are the principal radii of curvature of given surface, were already known at the end of the nineteenth century. They appear in the context of pseudospherical surfaces which are vastly explored and described in the literature.

Remaining forgotten for almost a century, surfaces of constant astigmatism reemerged recently in the systematic search for integrable classes of Weingarten surfaces. Under an adapted parametrization by lines of curvature, constant astigmatism surfaces correspond to solutions of the *constant astigmatism equation*

$$z_{yy} + \left(\frac{1}{z}\right)_{xx} + 2 = 0.$$

In the talk, an overview of new results regarding surfaces of constant astigmatism and constant astigmatism equation will be given. We construct a constant astigmatism surface from the pair of its complementary pseudospherical evolutes by purely algebraic manipulations and differentiation, i.e. using no integration. An extension of the famous Bianchi superposition principle then enables us to generate new surfaces of constant astigmatism and new solutions of constant astigmatism equation.

We also describe a connection between constant astigmatism surfaces and spherical *orthogonal equiareal patterns* with significance in two-dimensional plasticity.

Finally, we construct a pair of reciprocal transformations for the constant astigmatism equation and generate some new solutions from the known seeds. We discuss the meaning of reciprocal transformations on the level of the sine-Gordon equation and we also describe how the reciprocal transformation acts on an orthogonal equiareal pattern.

About relations of Church theses and principles of uniformizations in the intuitionistic set theory

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We prove the first result from the list of results given below: U is independent of CT in non type set theory with two kinds of variables ZFIC2. M is the Markov's Strong Principle, DCS is the Double Complement of Sets. All principles have parameters of any kinds and their formulations can be found in [1] and [2].

SUMMARY of RELATIONS

1) ZFIC2+M+DCS+CT $\nvdash$ U (new result);
2) ZFIC2+M+DCS+U! $\nvdash$ U (from 1) and 4));
3) ZFIC2+M+DCS+CT!+U $\nvdash$ CT (1980);
4) ZFIR2+CT! $\vdash$ U! (Swartz, 1984);
5) ZFIR2+U $\vdash$ U! (trivial);
6) ZFIR2+CT $\vdash$ CT! (trivial);
7) ZFIC2+M+DCS+U $\nvdash$ CT! (1980);
8) ZFIC2+M+DCS+U $\nvdash$ CT (from 7) and 6)).

The following problem is still open:

$$\text{ZFIC2 without extensionality} + \text{M} + \text{DCS} + \text{CT} \nvdash \text{U!}.$$

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On combinatorics of wreath product of finite symmetric inverse semigroups

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For a set X, let $\mathcal{I}(X)$ denote the set of all partial bijections on X. Clearly, it is a semigroup under natural composition law. This semigroup is called the *full symmetric inverse semigroup* on X. If $X = \{1, \ldots, n\}$, then the semigroup $\mathcal{I}(X)$ is called *the full symmetric inverse semigroup* of rank n and is denoted by $\mathcal{I}_n$.

Let S be a semigroup, (P, X) be a semigroup of partial transformations of the set X. Define S^{PX} to be the set of partial functions from X to semigroup S:

$$S^{PX} = \{f : A \to S |\, \mathrm{dom}(f) = A, A \subseteq X\}.$$

Given $f, g \in S^{PX}$, the product fg is defined in the following way:

$$\mathrm{dom}(fg) = \mathrm{dom}(f) \cap \mathrm{dom}(g), (fg)(x) = f(x)g(x) \text{ for all } x \in \mathrm{dom}(fg).$$

For $a \in P, f \in S^{PX}$, define f^a as:

$$(f^a)(x) = f(xa),\ \mathrm{dom}(f^a) = \{x \in \mathrm{dom}(a); xa \in \mathrm{dom}(f)\}.$$

Wreath product of semigroup S with the semigroup (P, X) of partial transformations of the set X is the set

$$\{(f, a) \in S^{PX} \times (P, X) \,|\, \mathrm{dom}(f) = \mathrm{dom}(a)\}$$

with composition defined by $(f, a) \cdot (g, b) = (fg^a, ab)$. We will denote the wreath product of semigroups S and (P, X) by $S \wr_p P$.

Wreath product of inverse semigroups is an inverse semigroup. One can recursively define the wreath product of any finite number of inverse semigroups.

In my talk I will present some combinatorial results (e.g. cardinality of semigroup, combinatorics of Green's relations, combinatorics of nilpotents) concerning the wreath product of finite inverse symmetric semigroups. Because of the recursive definition of the wreath product, one can obtain a number of recurrent formulae, containing different types of known combinatorial objects.

Embedded graphs on Riemann surfaces and beyond

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This talk is based on the joint works with Natalia Amburg and George Shabat.

The subject of the talk lies in the intersection of algebra, algebraic geometry, and topology, and produces new interrelations between different branches of mathematics and mathematical physics. The main objects of our discussion are so-called Belyi pairs and Grothendieck dessins d'enfants. Belyi pair is a smooth connected algebraic curve together with a non-constant meromorphic function on it with no more than 3 critical values. Grothendieck dessins d'enfants are tamely embedded graphs on Riemann surfaces. Their connections provide a new way to visualize absolute Galois group action, new compactifications of moduli spaces of algebraic curves with marked and numbered points, new way to visualize some classical objects of string theory, mathematical physics, etc. Introduction to the theory will be given including modern applications. In particular, we will discuss the generalized Tchebyshev polynomials and their geometry, visualization of the Galois group action and its invariants, Grothendieck dessins d'enfants on reducible curves, numerous examples.

Normality of maximal torus orbits closures in simple modules of algebraic groups

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Our problem belongs both to algebraic geometry and combinatorics. We investigate normality of torus orbits closures in simple modules of simple algebraic groups. Every module can be given by its set of weights, and the normality property of a torus orbit closure reads combinatorially as the Saturation Property of the corresponding set of torus weights: the semigroup generated by them coincides with the intersection of the cone and the group generated by the same points. We restrict ourselves to tori which are maximal tori in algebraic groups, and the torus action in the module is induced by the algebraic group action. For any simple algebraic group G with a maximal torus T and its simple module V, we are interested in the following question: is it true that for all v from V the closure of Tv is a normal variety? Combinatorially, we need to check whether all the subsets in the corresponding set of weights are saturated. Now the problem of classification of all such pairs (G,V) is completely solved (arXiv: 0806.1981, 1009.4724, 1105.4577). In the talk, I am going to overview the methods of proving the Saturation Property, to show how they are applied in the nicest cases, and explain how to construct a non-saturated subset for the infinite number of cases which do not appear in the classification.

On groups, critical with respect to a spectrum

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Let G be a finite group. The set of the element orders of G is called the *spectrum* and denoted by $\omega(G)$. An arbitrary quotient group H/N, where $N, H \leq G$ and $N \trianglelefteq H$, is called a *section* of G. As in [1], we call a group G *critical* with respect to its spectrum if every proper section of G has the spectrum different from $\omega(G)$.

It has been proven in [1] that the number of groups critical with respect to some fixed spectrum is always finite. V. D. Mazurov stated the following

Conjecture. *For any positive integer n there exists an integer $k > n$, such that there exist k pairwise non-isomorphic finite groups, critical with respect to some fixed spectrum.*

Theorem 1. *Suppose that s is a positive integer and*

$$\{p_{ij} \mid i = 1, \ldots, s, j = 1, 2\}$$

is a set of pairwise unequal prime numbers. Then there exist at least 2^s pairwise non-isomorphic finite groups, critical with respect to

$$\Omega = \{n_1 \ldots n_s \mid n_j \in \{1, p_{j1}, p_{j2}\}\}.$$

Theorem 1 implies that there is no such positive integer n that for any spectrum there exist at most n groups, critical with respect to this spectrum. Thus the subsequent conjecture is that if we consider only spectra of simple groups, then such integer is going to exist.

Our first object of study is the alternating group A_6.

Theorem 2. *Let G be a group, critical with respect to the spectrum $\omega(A_6) = \{1, 2, 3, 4, 5\}$. Then G is either A_6, or $K \leftthreetimes A_5$, where K is a natural module of order 16 for the group $SL_2(4) \simeq A_5$.*

The work was partially supported by RFFI Grant 13-01-00505.

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Periodic groups acting freely on Abelian groups

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Let π be a set of primes. Periodic group G is called a π-*group*, if all of its element orders are divisible only by primes from π. A group G *acts freely* on a non-trivial group V, and the action is called a *free action*, if $vg = v$ implies $v = 1$ or $g = 1$ for $v \in V$ and $g \in G$.

Our goal is to describe $\{2, 3\}$-groups acting freely on Abelian groups.

Locally cyclic group G is a group such that every finite subset of G is contained in a cyclic subgroup.

Quaternion group is a quaternion group of order 8 or generalized quaternion group. *Locally quaternion group* is a 2-group G such that every finite subset of G is contained in a quaternion subgroup.

Quaternion group of order 8 possesses an automorphism of order 3. The corresponding semidirect product of order 24 is isomorphic to the group $SL_2(3)$. Denote by $\tilde{S}_4$ the extension of a group of order 2 with a group S_4 of degree 4, which has a quaternion Sylow 2-subgroup.

Theorem 1. *Let G be a $\{2, 3\}$-group acting freely on an Abelian group. Then one of the following statements is true.*

1) *G is locally finite and isomorphic to one of the following groups*:
   - *locally cyclic group*;
   - *direct product of a locally cyclic 3-group with a locally quaternion group*;
   - *semidirect product of a locally cyclic 3-group R with a cyclic 2-group $\langle b\rangle$, where $b^2 \neq 1$ and $a^b = a^{-1}$ for every $a \in R$*;
   - *semidirect product of a locally cyclic 3-group R with a locally quaternion group Q, where $|Q : C_Q(R)| = 2$*;
   - *semidirect product of a quaternion group $Q_8 = \langle x, y\rangle$ of order 8 with a cyclic 3-group $\langle a\rangle$, where $x^a = y$*;
   - *group $\tilde{S}_4$.*
2) *G is not locally finite and all prime order elements of G generate a cyclic subgroup.*

Any of the groups mentioned can act freely on some Abelian group.

We give the full description of groups from i. 2 of Theorem 1.

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New properties of Riemann's zeta function

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Riemann's zeta function, studies already by Euler, is one on the most interesting and important object of investigations in Number Theory thanks to its relationship with the distribution of prime numbers. Famous Riemann's Hypothesis about the zeroes of this function remains open for more than a century and a half.

In the last years the speaker empirically discovered several new properties of the zeroes of Riemann's zeta function, including new relationship with prime numbers. These discoveries were made as a result of intensive computer calculations, part of which was performed on computers of Armenian National Grid Initiative Foundation.

More detailed information about this ongoing research can be found on `http://logic.pdmi.ras.ru/~yumat/personaljournal/artlessmethod`.

The Diamond Lemma and prime decompositions of global knots

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We prove a new version of the famous Diamond Lemma of M.H.A. Newman (1942) suitable for working with topological objects. Using it we prove several positive and negative results on prime decompositions of knots in thickened surfaces.

Groups with given element orders

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We plan to give a review of periodic groups with prescribed element orders. Here is one of the latest results in this topic.

Theorem 1. *(E. Jabara, D. V. Lytkina, V. D. Mazurov, not yet published) Let G be a $\{2,3\}$-group containing elements of orders 2 and 3 and no elements of order 6. Suppose that a period of every subgroup of G, generated by a couple of elements whose orders are at most 4, is a divisor of $72 = 2^3 \cdot 3^2$. Then G is either locally finite or is an extension of a 2-group by a 3-group containing a unique subgroup of order 3.*

Results of S. I. Adian [1] imply that there exist non locally finite groups which satisfy the conditions of Theorem 1.

Corollary 1. *Non-primary group of period 72 with no elements of order 6 is locally finite.*

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Simple modular Lie algebras

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It is now a fact that all finite-dimensional simple Lie algebras over an algebraically closed field of characteristic $p > 3$ fall into the following three families:

- Classical Lie algebras (modular analogs of the complex simple Lie algebras)
- Cartan type Lie algebras and their filtered deformations (thus are closely related to the four families of infinite-dimensional simple Lie algebras of Cartan)
- Melikyan algebras (two parametric family of simple Lie algebras which exist only for characteristic $p = 5$).

The aim of this talk is to give a comprehensive overview of simple Melikyan algebras, followed by a brief introduction to the classical and Cartan type simple Lie algebras.

Counting abelian subgroups in finitely generated metabelian groups

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Our aim is to present some of the results of our recent research with Prof. A.Yu. Ol'shanskii on abelian subgroups in finitely generated Abelian-by-polycyclic groups. The full classification of all abelian groups of that type was recently presented in [4], and it turned out to be useful for solution of problems in related areas. In particular, in [4] we were able to give a negative answer to a conjecture of G. Baumslag [1] about the number of abelian subgroups in metabelian group.

Here we would like to present an application to embeddings of groups. One of the most famous results on embeddings is the theorem of G. Higman, B. Neumann, H. Neumann [3] about embedding of any countable group H into a 2-generator group G. This theorem was a stimulus to extensive research in this direction, in particular, on embeddings when the group G is "close" to H in the sense that G satisfies a property provided that H satisfies it. If the group H is soluble, then G can also be selected to be a soluble 2-generator group [6]. And if the solubility length of H is n, then G can be a group of solubility length at most $n+2$ [6]. This limit in general is the best possible, since there are some examples of groups H of solubility length n, which cannot be embedded in any 2-generator group G of solubility length $n+1$. One of such examples is built by Ph. Hall in [2, Lemma 2]: the additive group of rational numbers $\mathbb{Q}$ (a soluble group of length 1, since it is abelian) cannot be embedded in any finitely-generated metabelian group (soluble group of length 2). Another example is built by B.H. Neumann and H. Neumann in [6, Lemma 5.3]: the quasicyclic group $C(p^\infty)$ (yet another abelian group) also does not possess such an embedding. Generalizing these we give a description for all abelian groups that can be embedded into 2-generator metabelian groups.

Using this criterion it is possible to built for any n a soluble group H of length n, which cannot be embedded into a 2-generator soluble group G of length $n+1$, although it can be embedded into a 2-generator soluble group G of length $n+2$ by [6]. Technics used in [4] can be modified for embeddings of ordered groups to give classifications for all the cases when an ordered rational group can be embedded into an ordered 2-generator rational group of solubility length 3 or 2 (metabelian group). This continues our research of [5] on embeddings of rational groups.

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Example of semi-Einstein submanifolds, different from the cone over sphere

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During the research of various classes of Ricci-semi-symmetric submanifolds in Euclidean spaces authors had geometrically described some classes of semi-Einstein submanifolds which outwardly represented cones over Einstein submanifolds, particularly cones over spheres. In all considered cases index of nullity of submanifolds was equal to the index of relative nullity. A question on whether there exist semi-Einstein submanifolds, different from cones over Einstein submanifolds, remained open.

In the present work normally flat submanifolds of codimension two are considered in Euclidean spaces with one multiple regular principal curvature vector and one nonzero singular principal curvature vector. Such submanifold automatically is semi-Einstein, has zero index of relative nullity, and its index of nullity is equal to 1. Totally integrable differential system is received, defining such submanifold, and its geometrical description is given. Internally such submanifold is locally isometric to a cone over a sphere, i.e. its metrics in some local coordinate system has the same form, as that of the cone over the sphere. However, from the point of view of ambient Euclidean space, it is not a cone over a sphere. In particular, it can be a direct product of hypersphere and some flat curve with nonzero curvature.

Universalization: Diophantine classification of simple Lie algebras and universal invariant volume of simple Lie groups

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Vogel introduced the Universal Lie algebra, with the motivations from knot theory, aimed “to construct a monoidal category which looks like the category of module over a Lie algebra and which is universal in some sense“ [1]. Delign’s et al. [2] approach of series of Lie algebras leads to a similar formulae, restricted to exceptional series of simple Lie algebras. These ideas lead to new interesting formulae both in Lie algebras and gauge theories. A typical result is the universal expression for dimensions of simple Lie algebras:

$$dim\,\mathfrak{g} \quad = \quad \frac{(\alpha-2t)(\beta-2t)(\gamma-2t)}{\alpha\beta\gamma} \tag{1}$$

where projective (Vogel's) parameters α, β, γ define a point on a Vogel's plane, which is the projective plane factorized w.r.t. the all permutations of three projective parameters. Expression (1) gives dimensions of simple Lie (super)algebras at special points at Vogel's plane, given in a table in [1], e.g. $A_{N-1} \sim (-2, 2, N), E_8 \sim (-2, 12, 20)$, etc.

One can try to "universalize" the theory of simple Lie algebras and groups, expressing it on the language of Vogel's parameters. In this report we present development of this approach mainly based on a universal expression for character of adjoint representations of simple Lie algebras at line $x\rho$ (ρ is a Weyl vector in roots space), derived in [3]:

$$f(x) = \chi_{ad}(x\rho) = \frac{\sinh(x\frac{\alpha-2t}{4})}{\sinh(x\frac{\alpha}{4})}\frac{\sinh(x\frac{\beta-2t}{4})}{\sinh(x\frac{\beta}{4})}\frac{\sinh(x\frac{\gamma-2t}{4})}{\sinh(x\frac{\gamma}{4})}, \; t = \alpha+\beta+\gamma \tag{2}$$

We seek all points on Vogel's plane, for which this character is regular on a finite x plane, as is the case for simple Lie algebras. We find [4] that this requirement leads to a seven Diophantine equations on three integer variables. Each solution of each of these equations corresponds to some simple Lie (super)algebra, or one of 50 similar, in some sense, objects, among which is the $E_{7\frac{1}{2}}$ Lie algebra [5]. All equations are of the form knm =(second order polynomial over k, n, m). We present a complete lists of solutions of these equations. For example, one of these equations is $\frac{2}{k} + \frac{2}{n} + \frac{2}{m} = 1$ (multiplied on knm) which contains simple Lie (super)algebras: A_{N-1} at $(k, n, m) = (-N, N, 2)$, D_4 at $(3, 3, -6)$, E_6 at $(4, 3, -12), E_8$ at $(5, 3, -30)$, and $D_{2,1,\lambda}$ at $(0, 0, 0)$.

We also consider the volume of compact simple simply connected Lie groups w.r.t. the Cartan-Killing metric and prove that it has a universal representation [6, 7]:

$$Vol(\alpha, \beta, \gamma) = (2\pi)^{dim\,\mathfrak{g}} \exp\left(-\int_0^\infty dx \frac{f(x) - dim\,\mathfrak{g}}{x(e^x-1)}\right) \tag{3}$$

with normalization $\alpha + \beta + \gamma = 1$. This is connected with multiple (4-ple) Barnes gamma-function and has an applications in Chern-Simons and knots theories [6].

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Algebras with hyperidentities of lattice varieties

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A Boolean quasilattice is an algebra with hyperidentities of the variety of Boolean algebras. In this talk we give a general characterization of Boolean quasilattices with two binary and one unary operations. A functional representation of finitely generated free Boolean quasilattices with two binary, one unary and two nullary operations is obtained.

A De Morgan quasilattice is an algebra with hyperidentities of the variety of De Morgan algebras. We give a structural result on De Morgan quasilattices and a functional representation of the free n-generated De Morgan quasilattice with two binary and one unary operations.

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Fixed point theorems for quantum-lattices

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Fixed point theorems are widely used in functional analysis, in particular, to prove the existence of solutions of operator equations on classical spaces. We prove a few fixed point theorems for quantum-lattices (or q-lattices), which extend Tarski's lattice-theoretical fixed point theorem. The algebra, $(L; \wedge, \vee)$, with two binary operations is called q-lattice, if it satisfies the following identities: 1. $a \wedge b = b \wedge a, a \vee b = b \vee a$ (commutativity); 2. $a \wedge (b \wedge c) = (a \wedge b) \wedge c, a \vee (b \vee c) = (a \vee b) \vee c$ (associativity); 3. $a \wedge (b \wedge b) = a \wedge b, a \vee (b \vee b) = a \vee b$ (weak idempotence); 4. $a \wedge (b \vee a) = a \wedge a, a \vee (b \wedge a) = a \vee a$ (weak absorption); 5. $a \wedge a = a \vee a$ (equalization).

Any q-lattice corresponds to a quasiorder (both a reflexive and a transitive relation) θ, which is defined in the following manner:

$$a\theta b \leftrightarrow a \wedge b = a \wedge a \leftrightarrow a \vee b = b \vee b.$$

Let θ be a quasiorder on the set, $L \neq \emptyset$; then $E_\theta = \theta \cap \theta^{-1} \subseteq L \times L$ is an equivalence. The relation, θ/E_θ, which is induced from θ on the set, L/E_θ, in the following manner:

$$(A, B) \in \theta/E_\theta \leftrightarrow a\theta b, \text{ where } \forall a \in A, \forall b \in B \text{ and } A, B \in L/E_\theta,$$

is an order. Further, the order θ/E_θ is denoted by $\leq_\theta$, and the class of equivalence, which includes the element x, is denoted by $[x] \in L/E_\theta$.

Note that the quasiordered set, $(L; \theta)$, is a q-lattice, if for each two classes of the equivalences, $[a], [b] \in L/E_\theta$, exist both $inf([a], [b]) = [a] \wedge [b]$ and $sup([a], [b]) = [a] \vee [b]$, i.e. if $(L/E_\theta; \leq_\theta)$ is a lattice.

For example, the $(Z \setminus \{0\}; \wedge, \vee)$ (where $x \wedge y = (|x|, |y|)$ and $x \vee y = [|x|, |y|]$, for which $(|x|, |y|)$ and $[|x|, |y|]$ are the greatest common divisor (gcd), and the least common multiple (lcm) of $|x|$ and $|y|$) is a q-lattice, which is not a lattice, since $x \wedge x \neq x$ and $x \vee x \neq x$. The quasiorder relation, corresponding to the q-lattice $(Z \setminus \{0\}; \wedge, \vee)$, is the relation of divisibility on $Z \setminus \{0\}$.

We assume that the presented fixed point theorems for q-lattices also can be used to solve operator equations on non-classical surfaces (manifolds).

Spectral data for several matrices and multivariate analogs of characteristic functions

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We consider p unitary matrices of size $\alpha+k$ determined up to a common conjugation by a unitary matrix of size k. There is a natural multiplication in such data imitating multiplication of operator colligations (nodes). We construct analogs of Livshits characteristic functions (transfer-functions) for this case. This functions are holomorphic, they map matrices of order p with norm ≤ 1 to matrices of order $p\alpha$ with norm ≤ 1 and also map unitary matrices to unitary matrices.

On infinite dimensional quadratic forms admitting composition

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It is not much known about the composition of infinite dimensional quadratic forms. It is proved in [1] the existence of the composition for positive quadratic forms of arbitrary infinite dimension. Analogous result by another method is received in [2] for positive quadratic form of countable dimension. It turns out that the construction given in [2] can be extended over other infinite dimensional quadratic forms.

Let F be an arbitrary field with unit element 1 and with characteristics $\neq 2$. The notion of infinite sign matrix can be defined analogous as in the case of finite dimension.

Let's consider the elements $\alpha_i \in F$ defined through the recurrent formula $\alpha_{P_k(l)} = \alpha_k \alpha_l$ by all primary value $\alpha_0 = 1, \alpha_1, \alpha_2, \alpha_4, \alpha_8 = \pm 1$. We will consider the following infinite quadratic forms $Q(x) = \sum \alpha_i x_i^2$. Then the quadratic form $Q(x)$ has the following representation:

$$\begin{aligned} Q(x) = x_0^2 + \alpha_1 x_1^2 + \alpha_2 x_2^2 + \alpha_1 \alpha_2 x_3^2 + \alpha_4 x_4^2 + \\ + \alpha_1 \alpha_4 x_5^2 + \alpha_2 \alpha_4 x_6^2 + \alpha_1 \alpha_2 \alpha_3 x_7^2 + \alpha_8 x_8^2 + \\ + \alpha_1 \alpha_8 x_9^2 + \cdots + \alpha_1 \alpha_2 \alpha_3 \alpha_4 x_{15}^2 + \alpha_1 x_{16}^2 + x_{17}^2 + \ldots \end{aligned}$$

Theorem 1. *For any infinite sign matrix S the quadratic form Q admits some composition.*

Let $A(\alpha_1, \alpha_2, \alpha_4, \alpha_8; S)$ is the linear normed algebra defined by the above mentioned composition.

Theorem 2. *In algebra $A(\alpha_1, \alpha_2, \alpha_4, \alpha_8; S)$ any left shift operator Φ_a, $\Phi_a(b) = a.b$ is a bijection if and only if $Q(a) \neq 0$. Besides, if S is a Unital sign Matrix, then the algebra A has a left identity element.*

Let F be an arbitrary ordered field containing all square roots of its positive elements.

Theorem 3. *For any finite dimensional composition algebra A with unit element over the field F there exists an infinite dimensional normed algebra of type $A(\alpha_1, \alpha_2, \alpha_4, \alpha_8; S)$ with left identity element, containing A as subalgebra.*

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Fundamental groups of spaces of trigonal curves

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The braid group can be considered as the fundamental group of the space of complex polynomials with distinct roots. Natural generalizations of the braid group are the fundamental groups of spaces of nonsingular hypersurfaces on algebraic varieties. They appears, for example, when we try to generalize to bigger dimensions the method of braids in the theory of real algebraic curves. We make a first step in this direction.

Let $q : \Sigma_k \to \ P^1$ be a complex rational ruled surface (Hirzebruch surface) with the exceptional section s, $s^2 = -k < 0$. A *trigonal curve* is a curve $A \subset \Sigma_k$ disjoint from $s(P^1)$ and with the restriction $q : A \to P^1$ of degree 3. The curve A can be determined by the Weierstrass equation $y^3 + b(x_0, x_1)y + w(x_0, x_1) = 0$, where b, w are homogeneous polynomials of degrees $2k$, $3k$.

Let $d = 4b^3 + 27w^2$ be the discriminant in y of the equation of A. The function $j : \mathbf{CP^1} \to \mathbf{CP^1}, \mathbf{j = 4b^3/d = 1 - 27w^2/d}$, is *$j$-invariant of the curve A*. Suppose $t_1, \dots, t_r$ is the set of all imaginary critical values of j and let $St(j)$ be the star in $\mathbf{CP^1}$ with the center in ∞ and the ray ends in t_i (a ray may contain another one). The graph $\mathrm{T\Gamma(j) = j^{-1}}(\mathbf{RP^1} \cup \mathbf{St(j)})$ is the *tetratomic graph* of A. The pair $(\mathrm{T\Gamma(j), all\ the\ critical\ values\ of j})$ is the *Riemann data* of A.

Denote by $Trig_k$ the set of trigonal curves on Σ_k with the non-constant j-invariant. *The space RD_k of all Riemann data is $Trig_k/PGL(2, \mathbf{C})$.* Using the Lyashko-Looijenga mapping $LL\colon RD_k \to \mathbf{C}[\mathbf{t}]$, $LL(\mathrm{T\Gamma}, \{\mathrm{t_1, \dots, t_r}\}) = (t - t_1)\dots(t - t_r)$ we prove that *RD_k is a finite cell complex.*

A nonsingular curve $A \in Trig_k$ is *almost generic* if it has no vertical flexes (i.e. the roots of the discriminant d are simple).

For the space $NSing_k \subset Trig_k$ of nonsingular curves and for its subspace $AlGen_k$ of almost generic curves the fundamental groups (for $k = 1$) and their images in the spherical braid group (for any k) have been calculated.

On the topology of real decomposable algebraic curves

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The talk gives a survey of the results on the problem of degree n decomposable algebraic curves topological classification in the real projective plane under the following conditions: i) each cofactor is an M-curve; ii) cofactors are in general position; iii) every two cofactors have the maximal number of common real points and these points belong to the same branch of each cofactor.

This problem belongs to the topic of the first part of the Hilbert 16th problem. The first non-trivial case of $n = 6$ was investigated in [1]. For $n \geq 8$ the complete solution of the problem is unreal since the number of types is very big. For $n = 7$ the solution is now almost completed, mainly due to efforts of A. Korchagin, S. Orevkov, E. Shustin, and the author in 1985 – 2012. Namely, the classification of affine M-sextics (i.e. unions of a line and a quintic) was completed in [2]. The classification of unions of a conic and a quintic was started in [3] and completed in [4] (if the common points of the cofactors lie on ovals) and in [5] (if they lie on odd branch of quintic). The arrangements of a plane real quintic curve with respect to a pair of lines were classified in [5] – [7]. The classification of unions of a quartic and a cubic with common points on ovals was obtained in [8]. The most difficult case – the same when common points are on odd branch – was being considered in [9],[10] and now it is almost completed (unpublished).

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Algebraic models of synchronized switch schemes

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By *the two-dimensional synchronized switch scheme* we mean n^2 identical *circular m-positional switches* arranged as elements of a square matrix of the order $n \times n$. This scheme is synchronized in the following sense: if one turns the switch with the number (i, j) then all switches in i-th string and j-th column automatically turn in the same way.

The *state* of the scheme is defined as a matrix of positions of all their switches. The *control problem* for this scheme is: starting with some initial state, obtain the desired final state. We will say that the scheme is *controllable*, if for any initial and final states there is a corresponding sequence of manipulations.

The *three-dimensional synchronized switch scheme* is defined similarly: switches are arranged in a cubic matrix of the order $n \times n \times n$. The synchronization is described as follows: if one turns on the switch with the number (i, j, k) then all switches with numbers $(i, j, *)$, $(i, *, k)$ and $(*, j, k)$ turn similarly.

Problem. *Find a* controllability criteria *(a relation between m and n) for these schemes; under the controllability condition, for given initial and final states find an explicit corresponding sequence of manipulations.*

Theorem 1. *1) The 2-dimensional scheme is controllable iff the number m is relatively prime with the numbers $n-1$ and $2n-1$.*

2) The 3-dimensional scheme is controllable iff $n > 2$ and the number m is odd and relatively prime with the numbers $n-2$, $n-1$, and $3n-2$.

Explicit solutions are based on the inversion of special linear transformations on $Mat_n^2(\mathbb{Z}_m)$ and $Mat_n^3(\mathbb{Z}_m)$ (modules of square and cubic matrices of the order n over the ring $\mathbb{Z}_m$ of residuals *modulo* m resp.): for 2-dimensional case $F(X) := -X + UX + XU$ (U consists of units); for 3-dimensional case $F(X) := V \circ X + X \circ V - X \times V - 2X$ (V consists of units, $\circ$ is the *frontally fiber* product and $\times$ is the *vertically fiber* product of cubic matrices respectively). In both cases the search for explicit solution is reduced to solving the equation of the form $F(X) = C$.

Theorem 2. *[2-dim] Under the controllability condition: for $m = 2$ we have $X = F^{-1}(C) = F(C)$; for $m > 2$ we have $X = F^{-1}(C) = (-1 + n' + n'')C + (n' + n'' - n'n'')F(C) - n'n''F^2(C)$, where $n' = (n-1)^{-1}$ and $n'' = (2n-1)^{-1}$ in the ring $\mathbb{Z}_m$.*

Theorem 3. *[3-dim] Under the controllability condition we have $X = F^{-1}(C) = a_0C + a_1F(C) + a_2F^2(C) + a_3F^3(C)$, where coefficients a_0, a_1, a_2 and a_3 are calculated by $(n-2)^{-1}$, $(n-1)^{-1}$, $(3n-2)^{-1}$, and 2^{-1} in the ring $\mathbb{Z}_m$.*

On the group $(C_3 \times C_3) \propto C_2$

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In the present paper we investigate up to isomorphism, there exist three non-abelian group of order 18.
1) the direct product of the symmetric group S_3 and of the cyclic group C_3.
2) the semidirect product of the cyclic group C_9 with the cyclic group C_2.
3) the semidirect product $(C_3 \times C_3)$ with C_2 .
The first and second cases we investigated in [1] and in [2]. In the present paper we describe Group $(C_3 \times C_3) \propto C_2$. We calculate the group automorphism of this group. It is interesting from point of view of description of order 3 hypergroups over group. The notion of hypergroup over group was introduced in [3]. The notion of completely reduced hypergroup over group was introduced in [4]. The completely reduced hypergroups of order 3 over group can be obtained only either from the symmetric group

S_3 or from the noncommutative groups of order 18. The order 3 hypergroups over group, arising from S_3, are investigated in [5]. In the paper we calculate all hypergroups of order 3 arising from this group.

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